Quantum State Tomography of 1000 Bosons: Reduced Density Matrices

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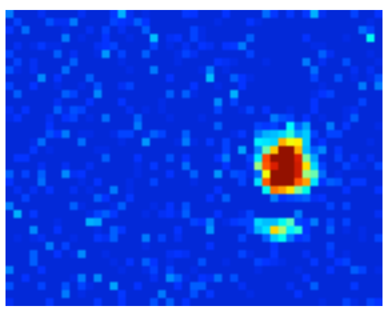
Motivation

BEC experiment → measurement → quantum state



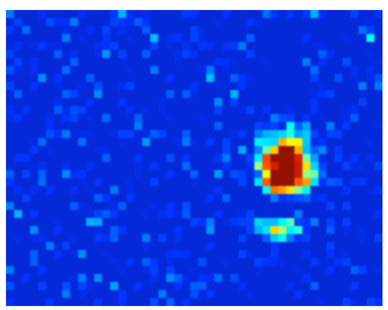
$N_\uparrow,\, N_\downarrow$? ρ_N

N ≈ 1000
two-level systems
Bose statistics

various directions,
several times

Motivation

BEC experiment → measurement → quantum state



$N_\uparrow, N_\downarrow$ → ? → ρ_N

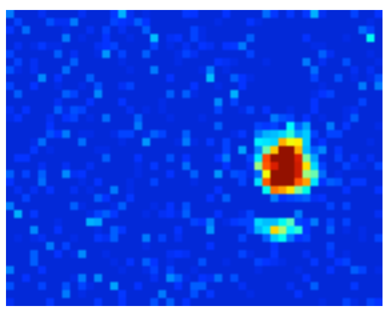
N ≈ 1000
two-level systems
Bose statistics

various directions,
several times

#measurements ≪ N^2
no state resolution

Motivation

BEC experiment | measurement | quantum state



$\longrightarrow$ $N_\uparrow, N_\downarrow$ $\overset{?}{\longrightarrow}$ ρ_N

N ≈ 1000
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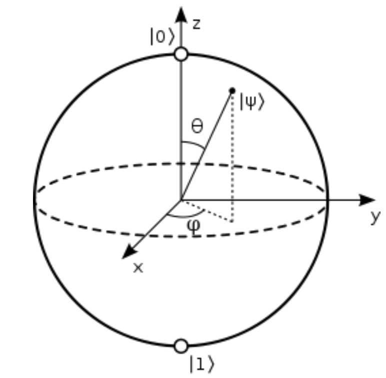
various directions,
several times

pseudo-spin
J=N/2

Angular Momentum Wigner Function



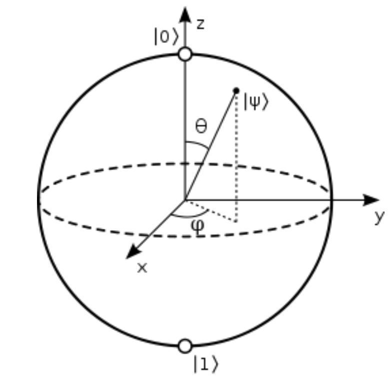
$$\rho_{k,q} = \sum_{M,M'} \langle J, M | \rho_N | J, M' \rangle \, t_{k,q}^{J,M,M'}$$

$$W_\rho(\psi) = \sum_{k=0}^{2J} \sum_{q} \rho_{k,q} Y_{k,q}(\psi)$$

Clebsch-Gordan coefficient

spherical harmonics

Angular Momentum Wigner Function



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$$W_\rho(\psi) = \sum_{k=0}^{2J} \sum_{q} \rho_{k,q} Y_{k,q}(\psi)$$

convolution

P-representation

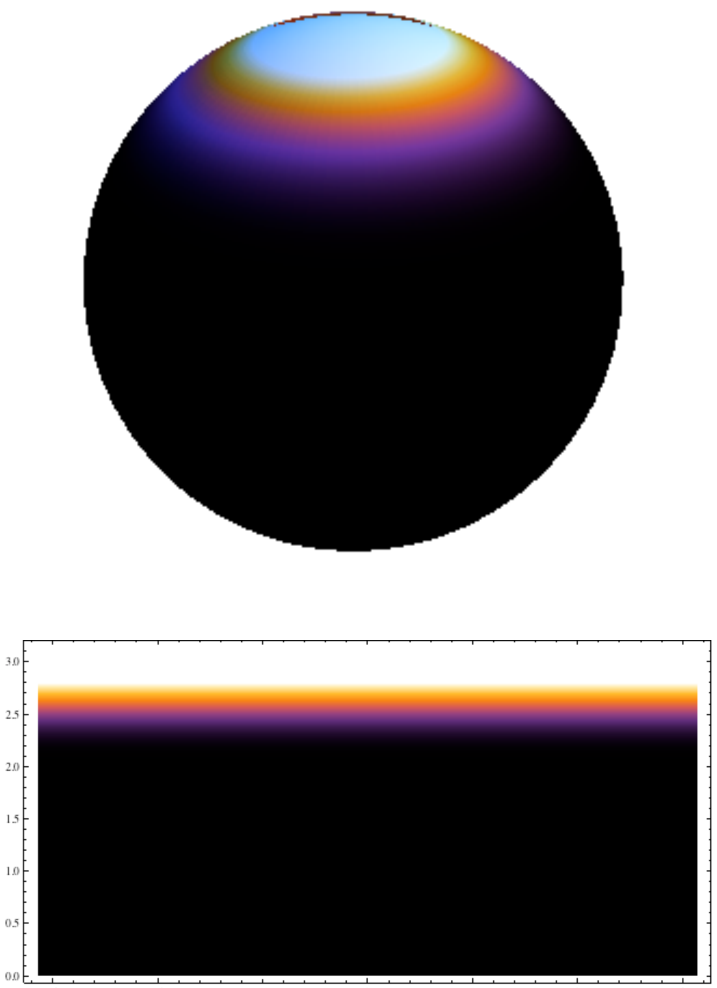
$$\rho_N = \int d\psi \, P_\rho(\psi) \, (|\psi\rangle\langle\psi|)^{\otimes N}$$

Q-representation

$$Q_\rho(\psi) = \langle\psi|^{\otimes N} \rho_N |\psi\rangle^{\otimes N}$$

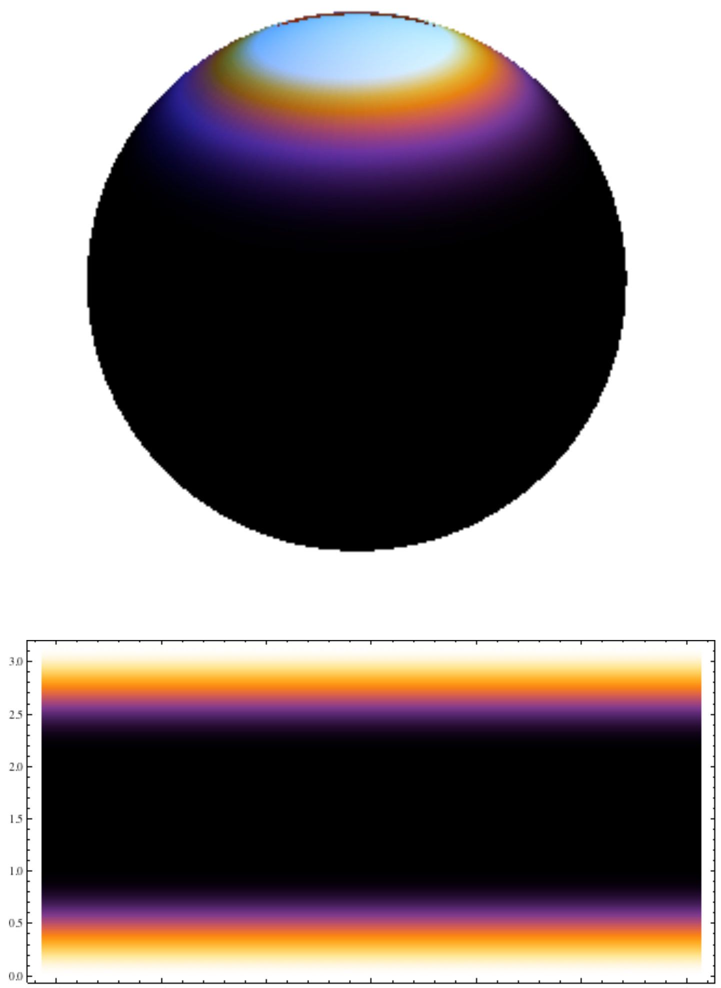
Examples

N=10



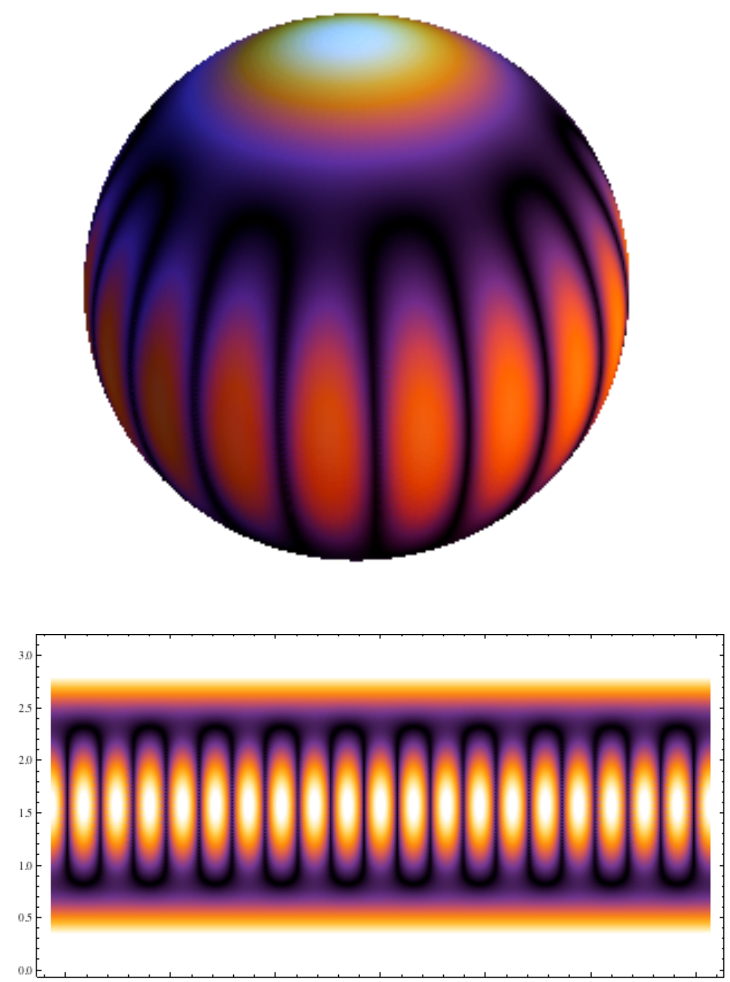
coherent state

$$|\uparrow \ldots \uparrow\rangle$$



coherent mixture

$$|\uparrow \ldots \uparrow\rangle\langle\uparrow \ldots \uparrow| \\ +|\downarrow \ldots \downarrow\rangle\langle\downarrow \ldots \downarrow|$$



cat state

$$|\uparrow \ldots \uparrow\rangle + |\downarrow \ldots \downarrow\rangle$$

Reduced Density Matrix

$$\rho_n = \mathrm{Tr}_{N-n}(\rho_N)$$

Wigner function coefficients

$$(\rho_{k,q})_{k \le k_{\max}}$$

cut off at $k_{\max} = n$

moments of one-body observables

$$\langle (O_1)^k \rangle_{\rho_N}$$

up to $k_{\max} = n$

Reduced Density Matrix

$$\rho_n = \mathrm{Tr}_{N-n}(\rho_N)$$

Wigner functio
coefficients

$$(\rho_{k,q})_{k \le k_{\max}}$$

cut off at $k_{\max} = n$

low frequency =
few body

ments of one-body
observables

$$\langle (O_1)^k \rangle_{\rho_N}$$

up to $k_{\max} = n$

Filtered Backprojection Method

Schmied and Treutlein, *New J. Phys* (2011)

measurement axes $\vec{e}^{(r)}$

measurement results $J^{(r)}, M^{(r)}$

$$\hat{\rho}^{(\mathrm{fbp})}_{k,q} = (2k+1)\frac{1}{R}\sum_{r=1}^{R}\Big(|J^{(r)}, M^{(r)}, \vec{e}^{(r)}\rangle\langle J^{(r)}, M^{(r)}, \vec{e}^{(r)}|\Big)_{k,q}$$

filter

backprojection

Filtered Backprojection Method

Schmied and Treutlein, *New J. Phys* (2011)

measurement axes $\vec{e}^{(r)}$

measurement results $J^{(r)}, M^{(r)}$

$$\hat{\rho}_{k,q}^{(\text{fbp})} = (2k+1)\frac{1}{R}\sum_{r=1}^{R} D_{q,0}^{k}(\vec{e}^{(r)})\, t_{k,0}^{J^{(r)},M^{(r)},M^{(r)}}$$

Wigner rotation matrix

Clebsch-Gordan coefficient

Properties

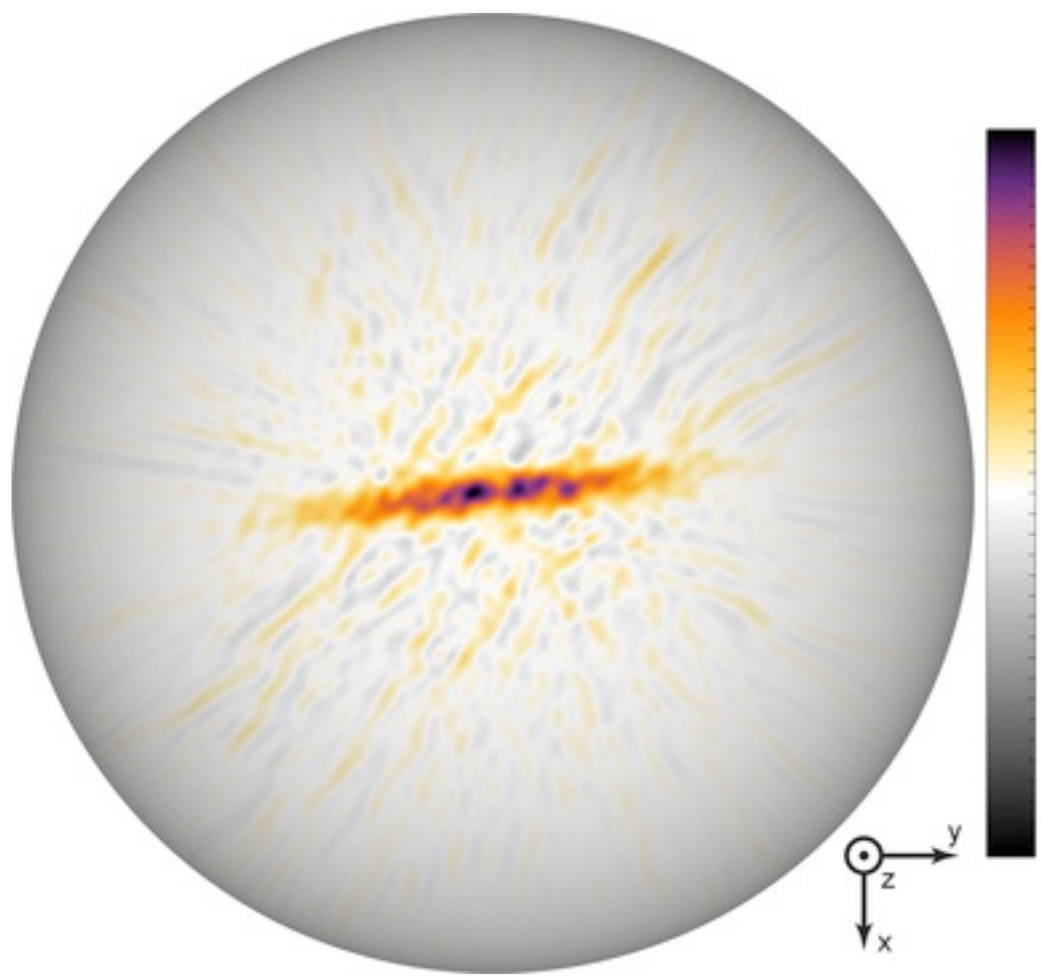
- asymptotically correct
- numerically stable
- for low k, insensitive to fluctuations in $J,\ M$ (do not need state resolution)

$$\hat{W}^{(\mathrm{fbp})}(\psi) = \sum_{k=0}^{k_{\max}} \sum_{q} \hat{\rho}^{(\mathrm{fbp})}_{k,q}\, Y_{k,q}(\psi)$$

Results

pseudo-spin squeezed state of N = 1250(45) ^{87}Rb atoms on an atom chip

Riedel *et al*, *Nature* (2010)

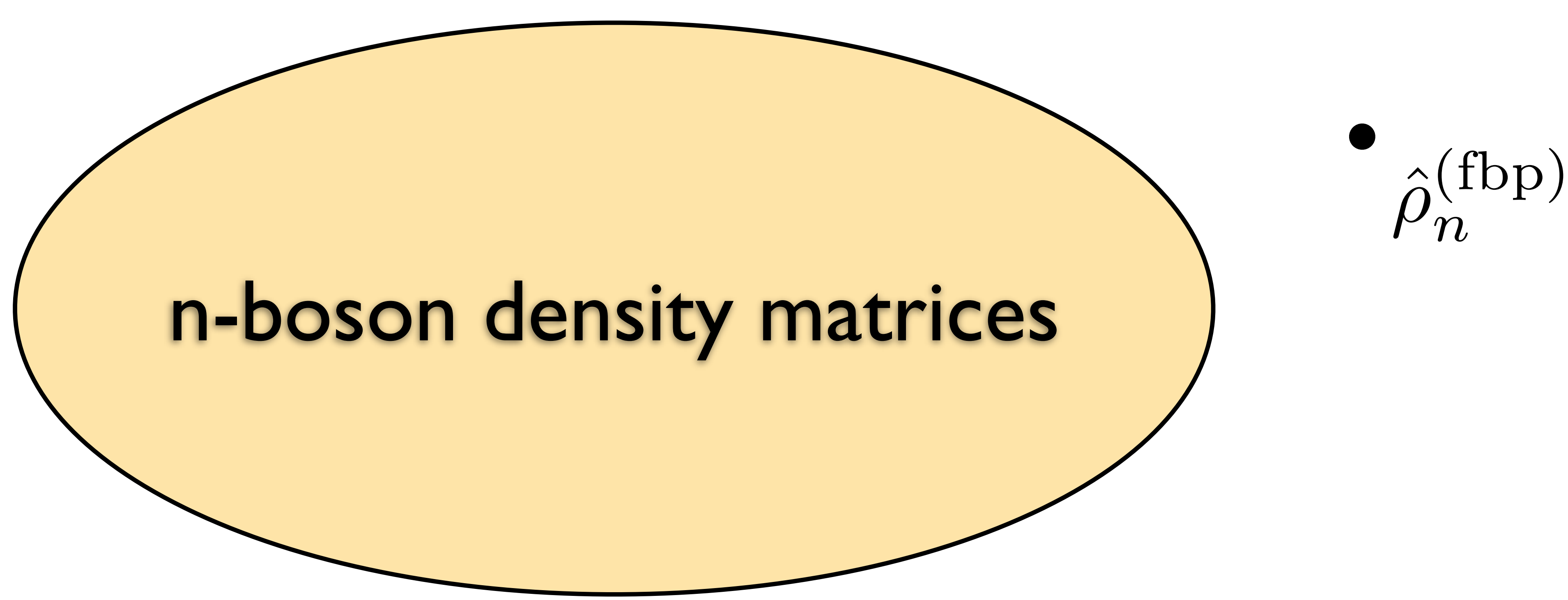


Schmied and Treutlein, *New J. Phys* (2011)

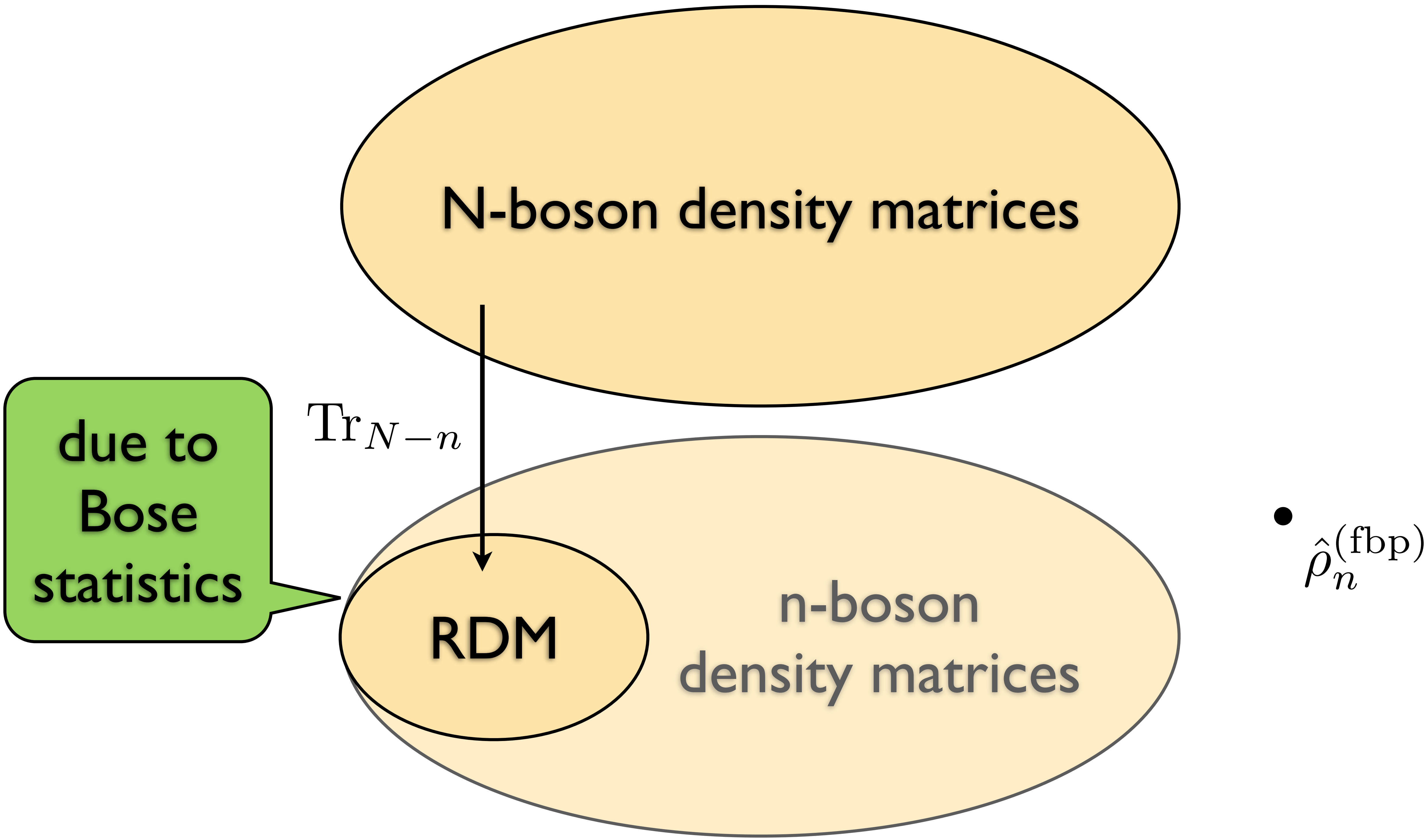
Positivity

reconstructed (reduced) density matrix can have <u>negative</u> eigenvalues

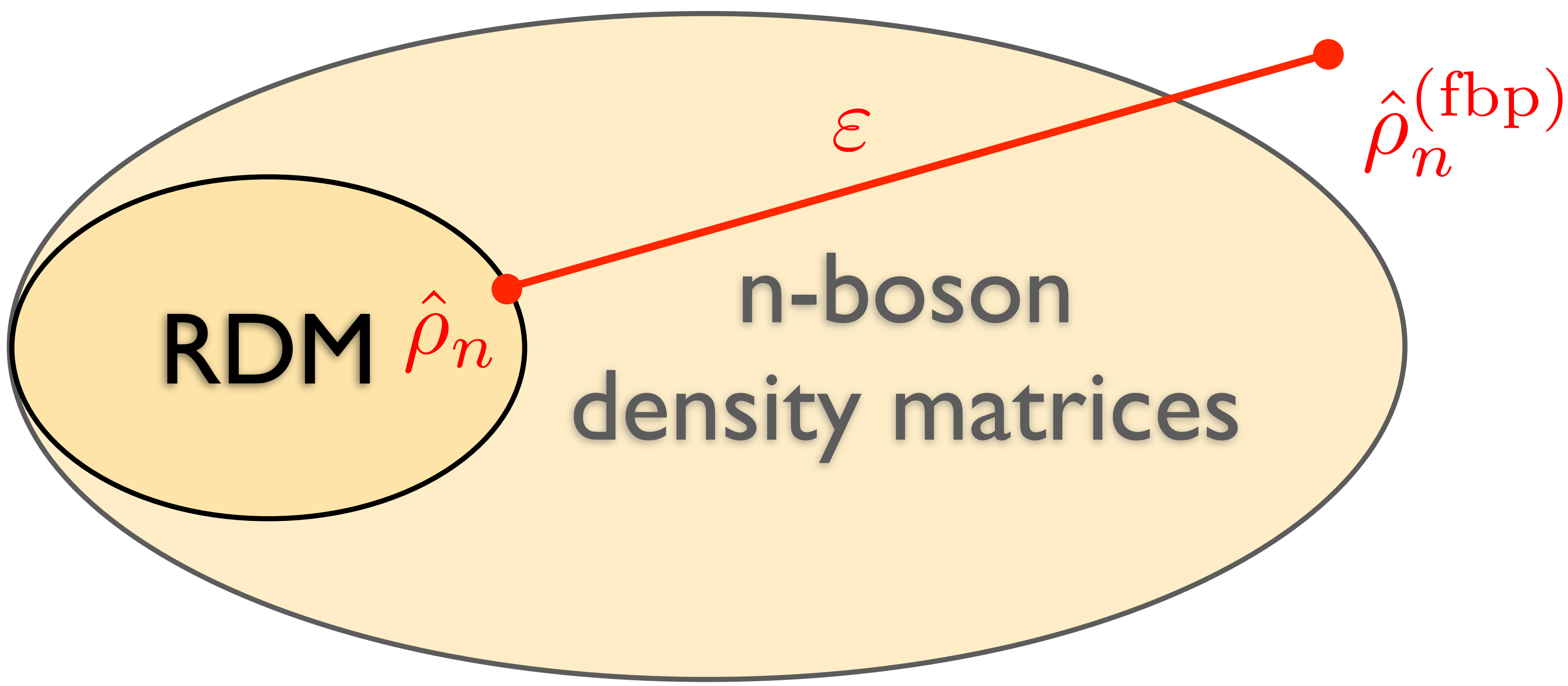
$n = k_{\max}$



N-Extendibility



Semidefinite Program for N-Extendibility



minimize $\varepsilon = \|\hat{\rho}_n - \hat{\rho}_n^{(\mathrm{fbp})}\|_1$

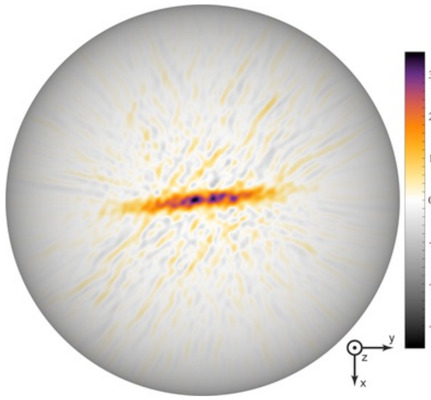
subject to $\exists \rho_N \geq 0 :$

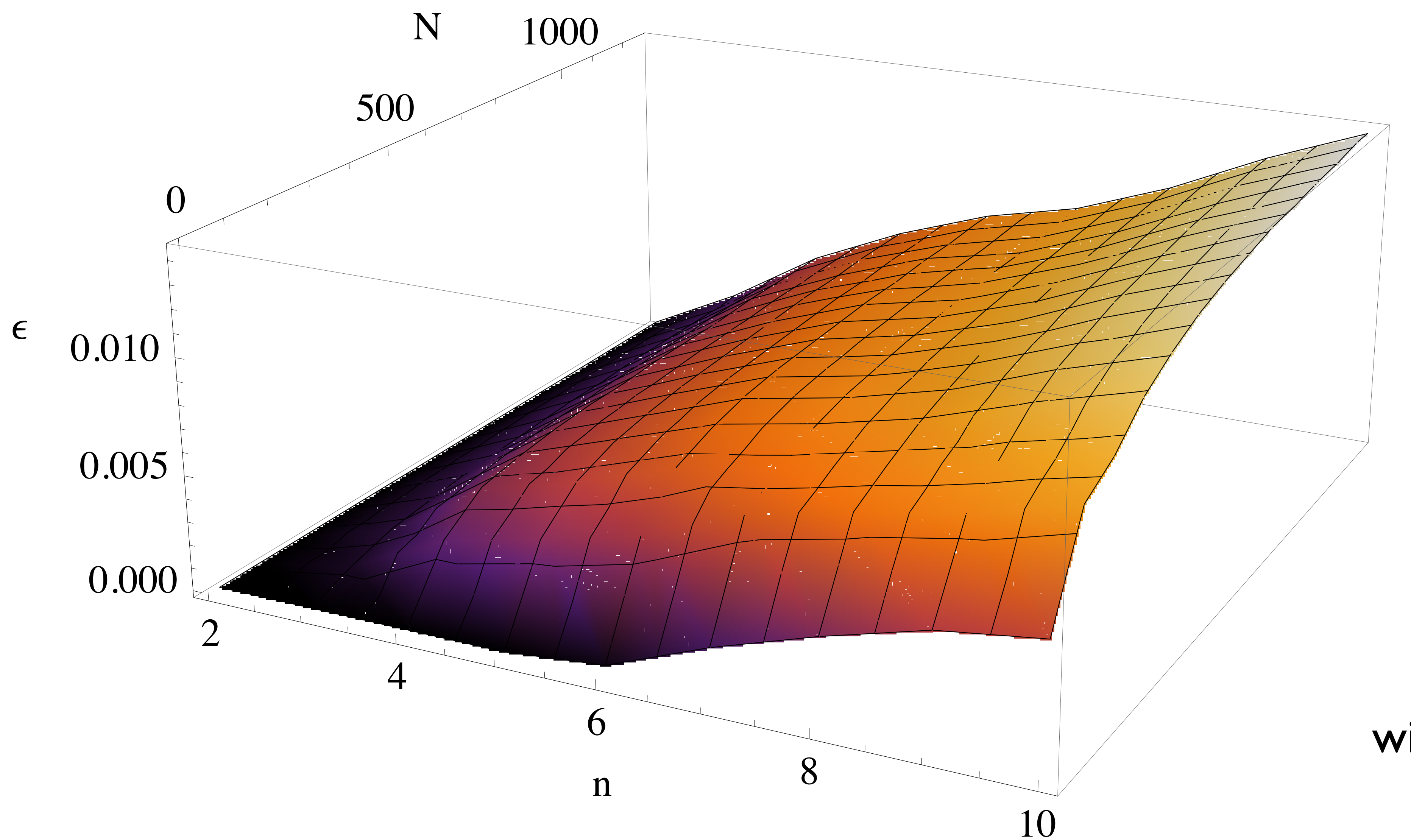
$\hat{\rho}_n = \mathrm{Tr}_{N-n}(\rho_N)$

efficient

Results

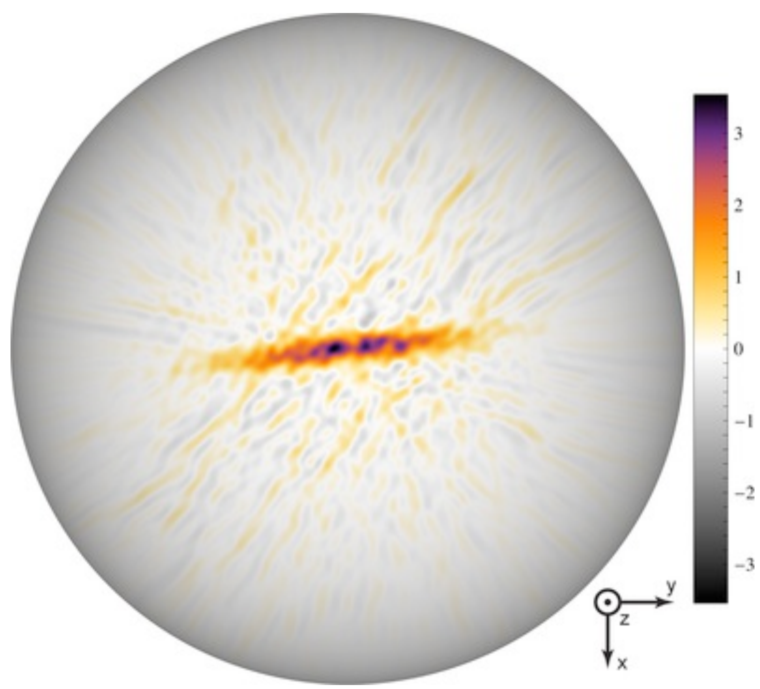
pseudo-spin squeezed state, N = 1250(45)

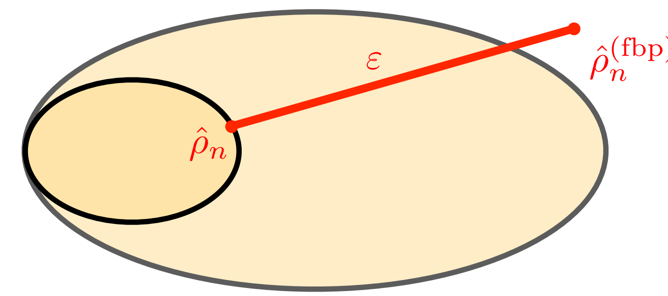


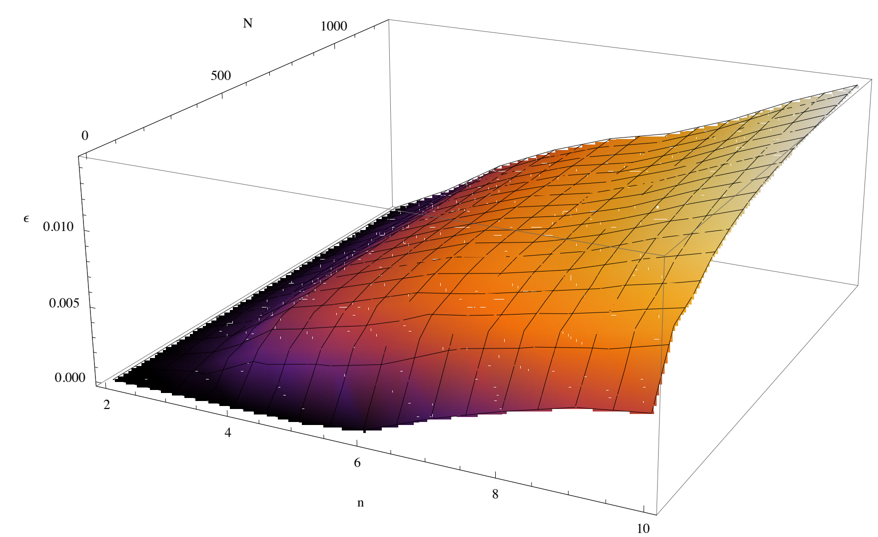


with S. Seehars
(2012)

Conclusion







Work in progress: MLE for moments, error bars