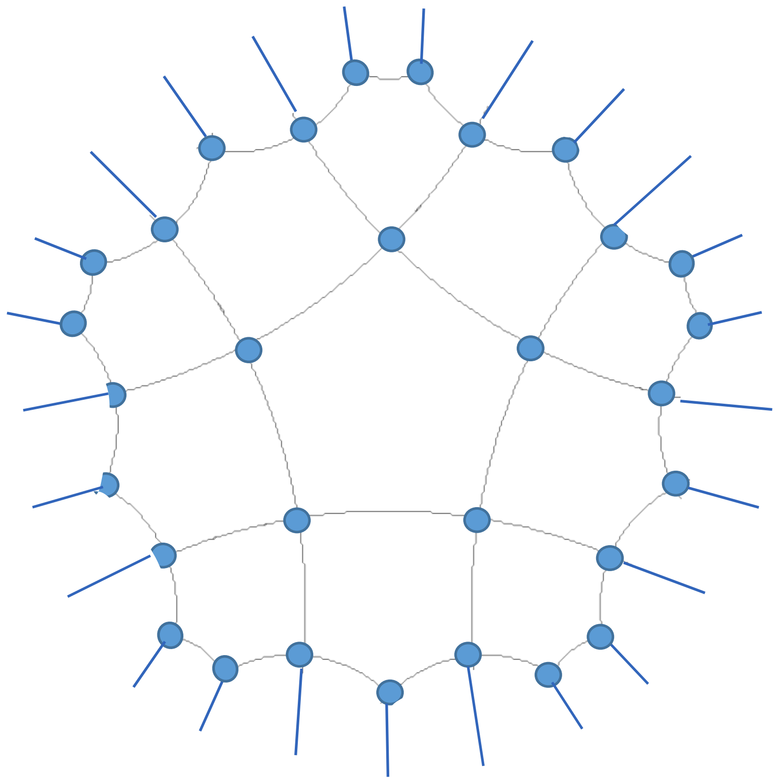


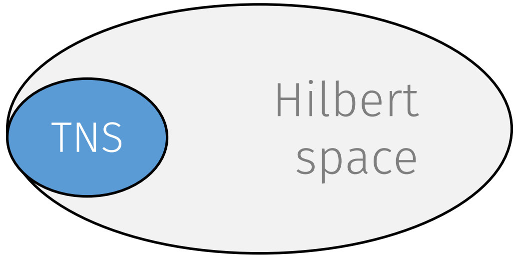
Quantum information in tensor networks

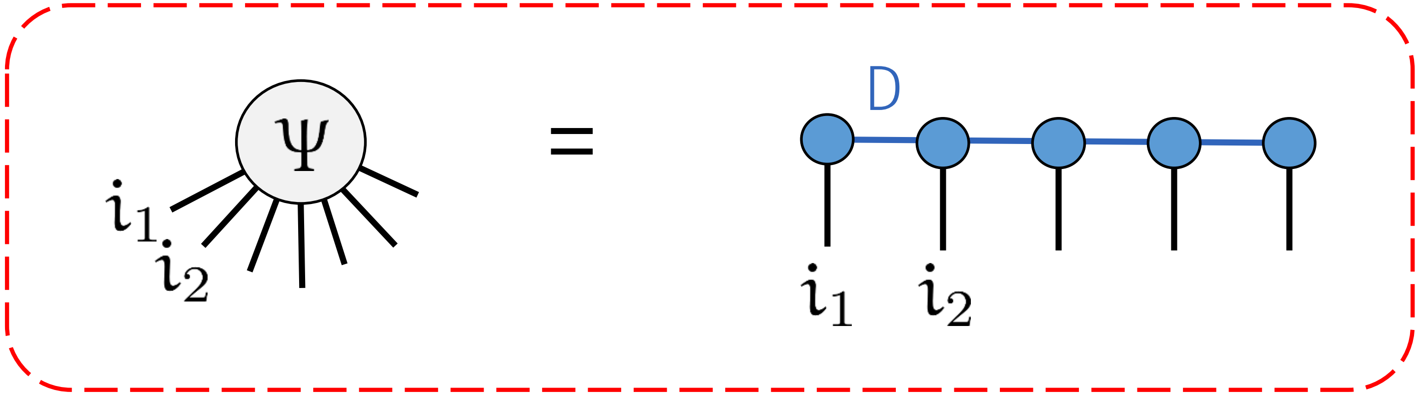
Michael Walter
Stanford University

IQC Colloquium, University of Waterloo – July 2016

Tensor network states

$$|\Psi\rangle = \sum_{i_1,\ldots,i_N} \boxed{\Psi_{i_1,\ldots,i_N}} |i_1,\ldots,i_N\rangle$$





example: matrix product state

efficient variational classes & useful theoretical formalism

ground states of quantum matter

quantum phases

topological order

RG circuits

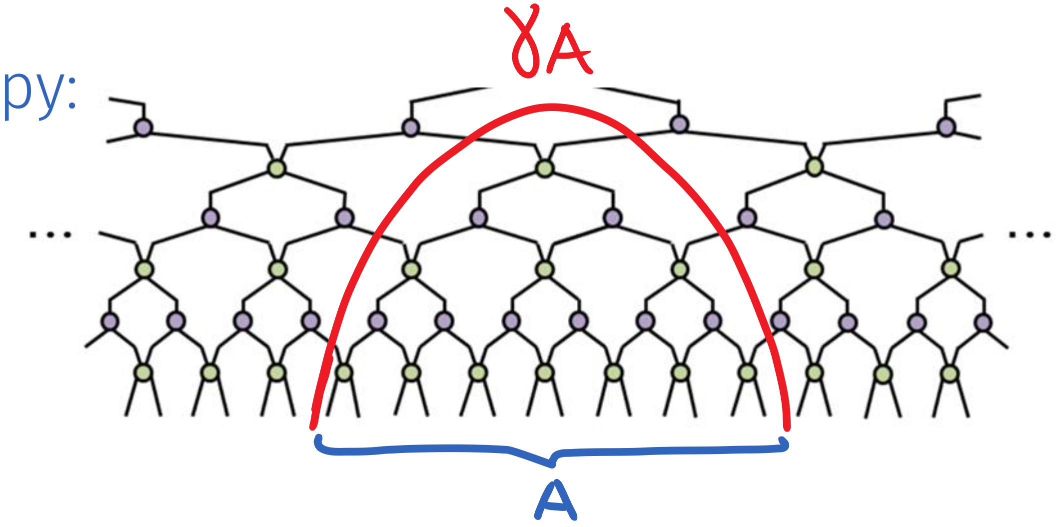
...

Tensor network kinematics (or: how to choose your corner in Hilbert space)

Fundamental bound on entanglement entropy:

$$S(A) \leqslant \log(D)\,|\gamma_A|$$

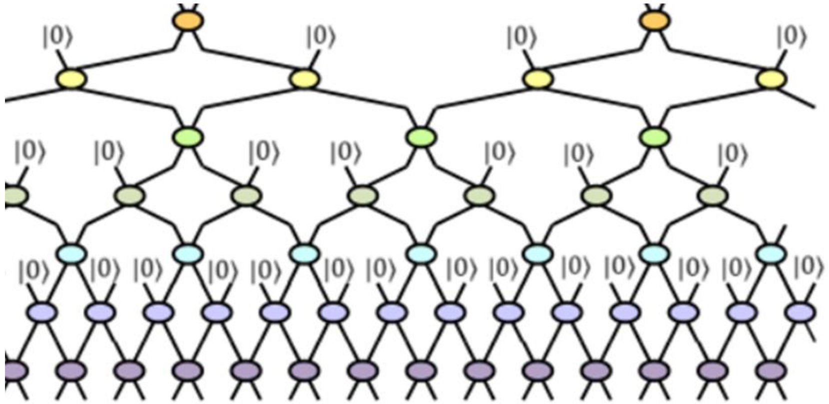
where $S(A) = -\operatorname{tr} \rho_A \log \rho_A$.



Bulk-boundary dualities: lift physics to the virtual level, e.g.

entanglement Hamiltonian
[Cirac et al]

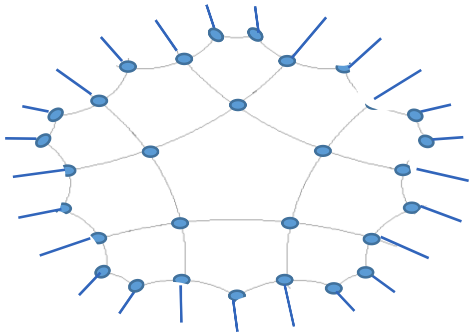
MERA as a RG circuit
[Evenbly-Vidal]



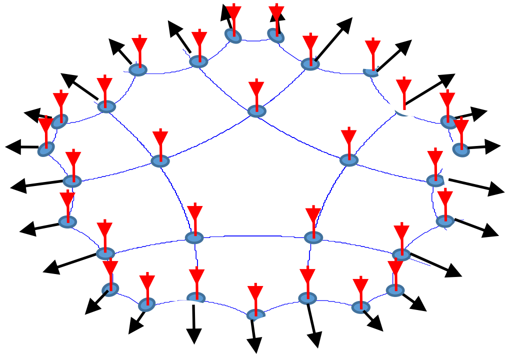
Figures from [Vidal]

Organization of quantum information? Properties of the bulk theory?

Plan for this talk

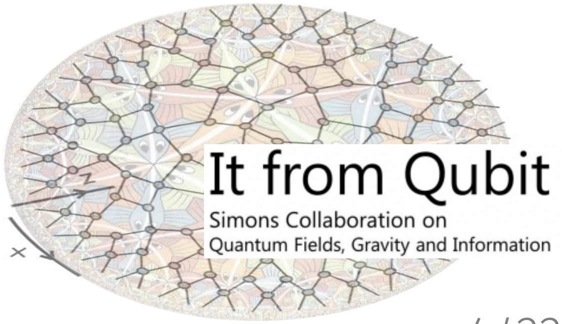


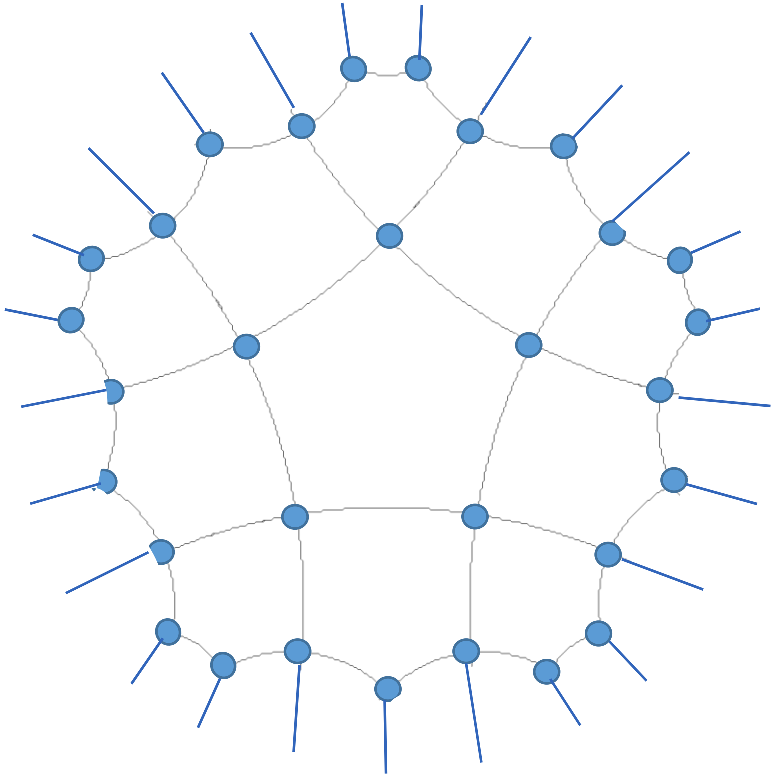
Random tensor networks and their curious entanglement structure. Two interpretations.



Bulk-boundary mappings as quantum error correcting holographic codes.

Throughout: Glances at the role of tensor network models in quantum gravity.



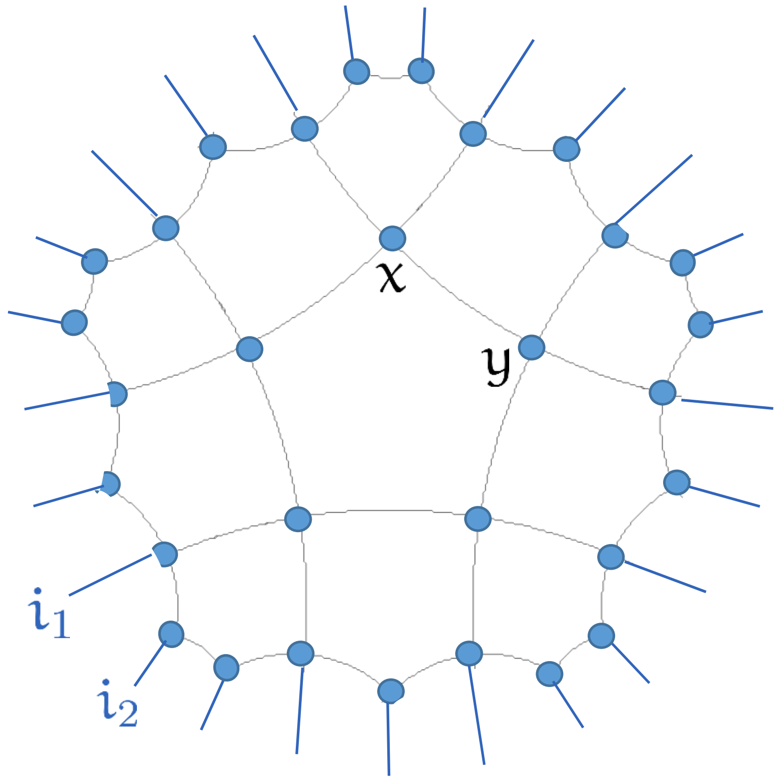


Random tensor networks

[Hayden-Nezami-Qi-Thomas-W.-Yang]

Random tensor network states

bond dimension D

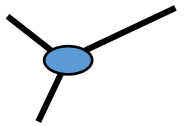


$$|\Psi\rangle = \Big(\bigotimes_{\langle x,y\rangle} \langle xy|\Big)\Big(\bigotimes_{x} |V_x\rangle\Big)$$

max. entangled states

$$|xy\rangle = \sum_{\mu=1}^{D} |\mu,\mu\rangle$$

random tensors

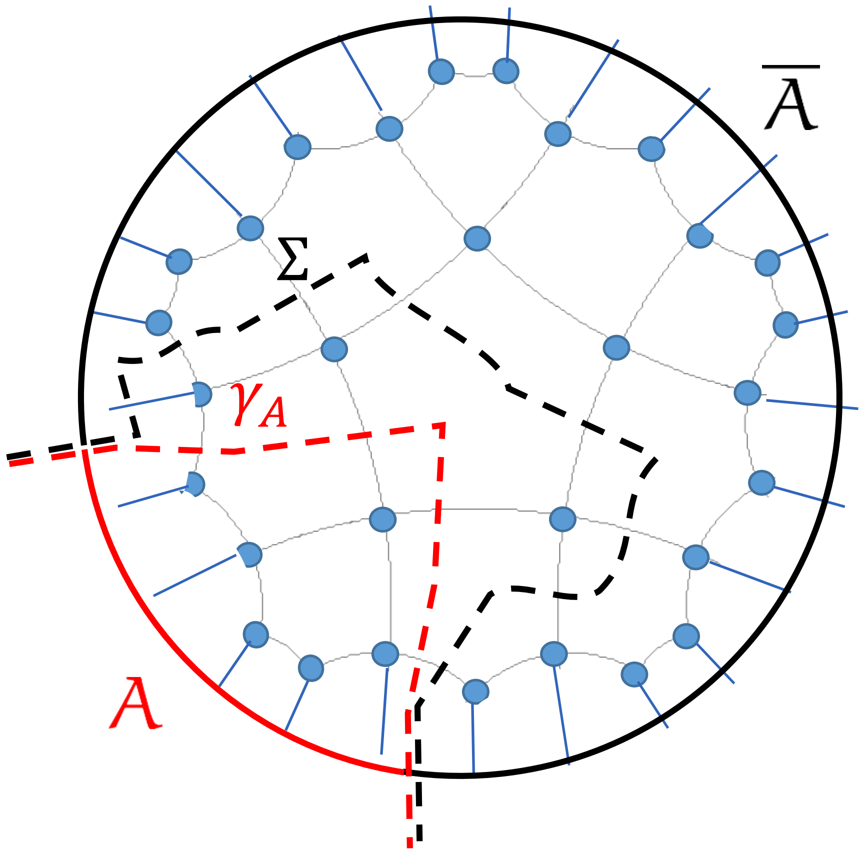


Random tensor network state on "boundary" of graph

Arbitrary lattice or graph. Tensors are chosen i.i.d. from Haar measure.

Entanglement entropy

$S(A) = -\operatorname{tr} \rho_A \log \rho_A$



Entanglement entropy in any tensor network:

$$S(A) \leqslant \log(D)\,|\gamma_A|$$

We will show that this is saturated in random tensor networks with large bond dimension D:

$$S(A) \simeq \log(D)\,|\gamma_A|$$

'Minimized area law' – also known as holographic or Ryu-Takayanagi entropy formula. Entropies are geometric!

Calculation of the lower bound

$$|\Psi\rangle = \left(\bigotimes_{\langle x,y\rangle} \langle xy|\right)\left(\bigotimes_{x} |V_x\rangle\right)$$

Lower-bound the Renyi-2 entropy:

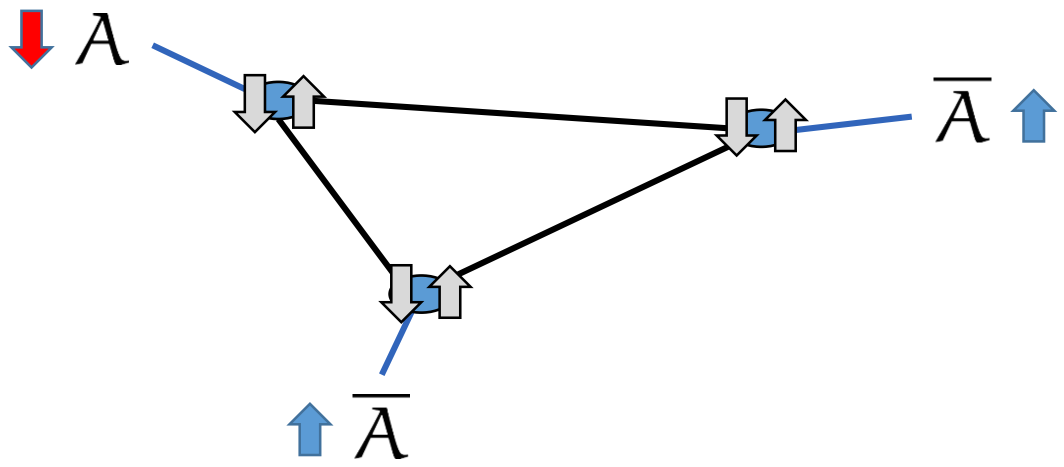
$$S_2(A) = -\log \operatorname{tr} \rho_A^2$$

(1) Swap trick:

$$\operatorname{tr} \rho_A^2 = \operatorname{tr}(\rho \otimes \rho)(F_A \otimes I_{\bar{A}})$$

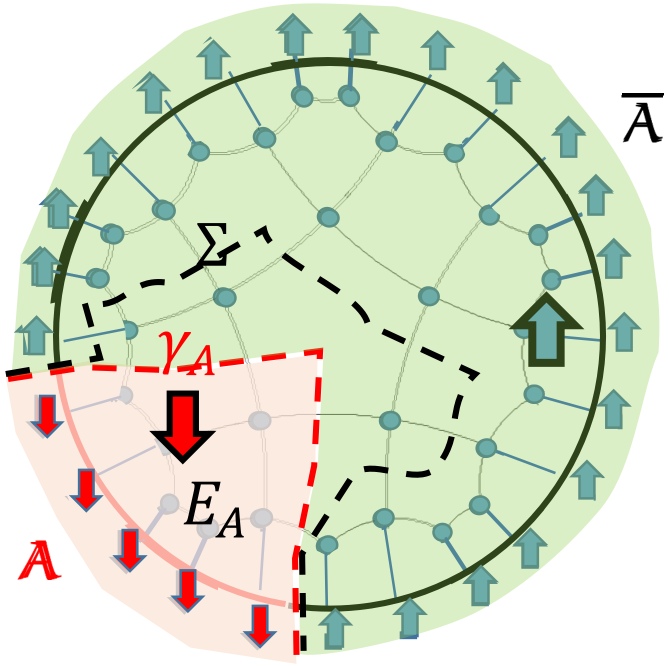
(2) Second moment of random tensors:

$$\overline{|V_x\rangle\langle V_x|^{\otimes 2}} \propto I_x + F_x$$



Interpretation 1: Ising model

$S_2(A) = -\log \mathrm{tr}\, \rho_A^2$



partition sum — 1/T — ferromagnetic Ising action

$$\overline{\mathrm{tr}\, \rho_A^2} \simeq Z_A = \sum_{\{s_x\}} e^{-\log D \times \frac{1}{2} \sum_{\langle x,y \rangle} (1 - s_x s_y)}$$

$$S_2(A) \simeq -\log Z_A \simeq \log(D)\, |\gamma_A|$$ large D / low T

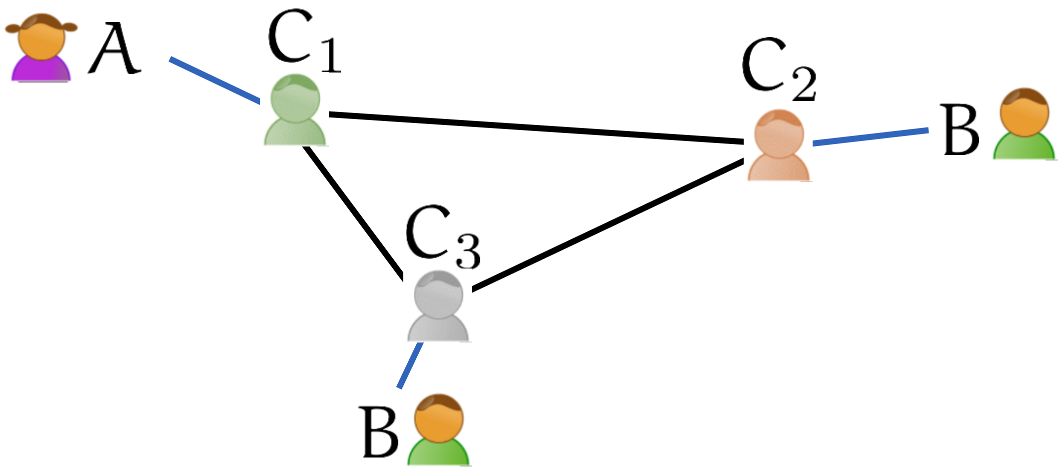
free energy, dominated by minimal energy cfg.

Thus the same is true for the entanglement entropy.

… - O(1) if multiple minimal domain walls. Can estimate D_{crit} from Ising physics [Onsager]!
Calculation only relied on second moments (2-design). Higher Renyis = higher moments.

Interpretation 2: entanglement distillation

[Horodecki-Oppenheim-Wi.], [Smolin-Verstraete-Winter], [Hayden-Dutil]



initial collection of Bell pairs

$$|\Psi_{AB}\rangle \propto \underbrace{\left(\bigotimes_{x} \langle V_x|\right)} \overbrace{\left(\bigotimes_{\langle x,y\rangle'} |xy\rangle\right)}$$

measurement in random basis optimal!
merges state w.h.p.

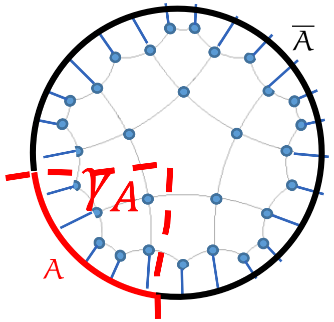
"Entanglement of assistance": How much entanglement can Alice and Bob distill with help of Charlies, by measuring & classically communicating results?

$$\lim_{n\to\infty} \frac{1}{n} E_{\text{assist}}(A^n : B^n) = \min_{I\subseteq\{1,2,3\}} S(AC_I) = \min_{\gamma_A} \log(D)\,|\gamma_A|$$

General mechanism for producing 'minimized area law' from area-law state!

Holographic entropy inequalities

$$S(A) = \log(D)\,|\gamma_A|$$



This entropy formula has rather remarkable properties.

It satisfies many nonstandard entropy inequalities, e.g. [Bao-Nezami-Ooguri-Stoica-Sully-W.]

$$I(A:B) + I(A:C) \leqslant I(A:BC)$$

[Hayden-Headrick-Maloney]

This monogamy inequality does not hold for general states.

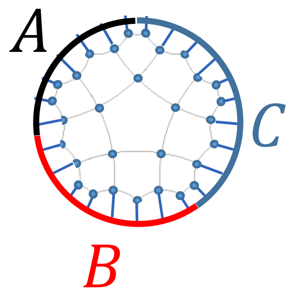
Indeed, correlations are not in general monogamous (unlike q. entanglement):

I(A:B) = S(A) + S(B) − S(AB) is the mutual information. It is zero for product states $\rho_A \otimes \rho_B$.

Multipartite entanglement in tensor networks

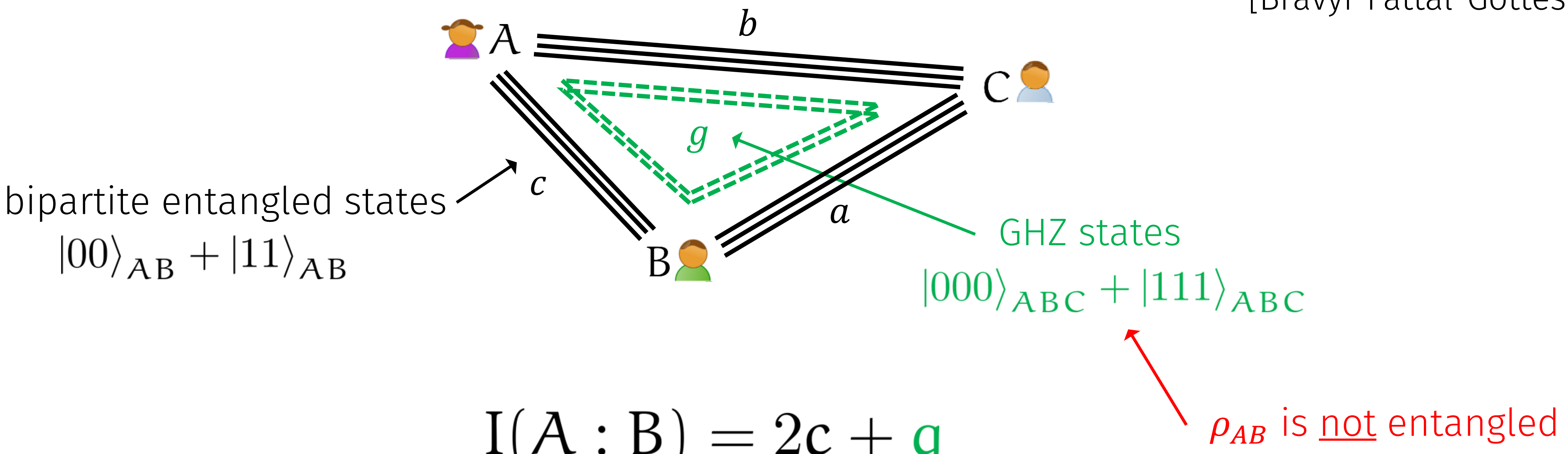
Does I(A:B) in fact measure entanglement in random tensor networks?

Study tripartite entanglement!



We restrict to stabilizer states. Any tripartite stabilizer state is of the form

[Bravyi-Fattal-Gottesman]

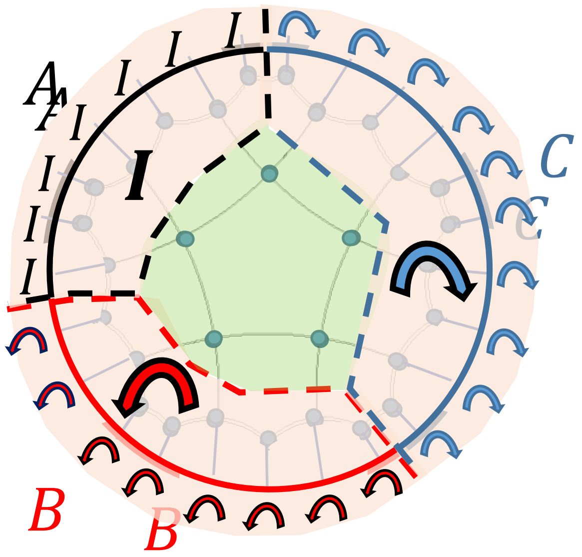


$$I(A:B) = 2c + g$$

Multipartite entanglement in random TNs

[Nezami-W.]

In random stabilizer networks there is only little tripartite entanglement:



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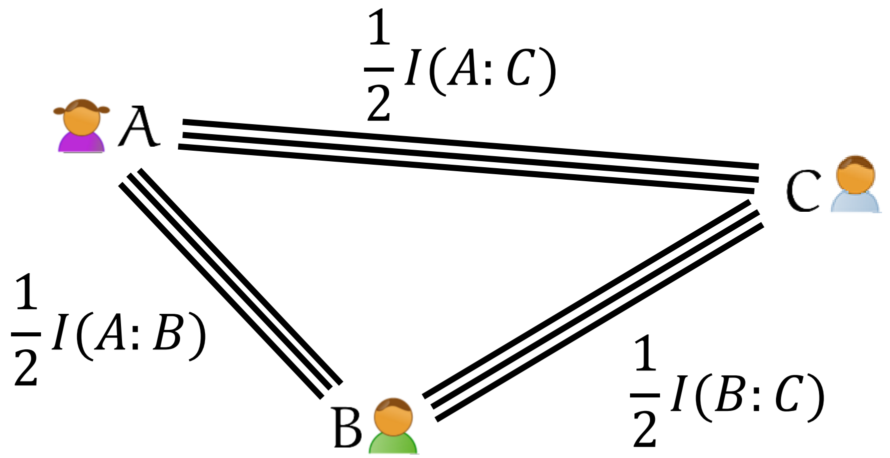


# of GHZs → $tr\left(\rho_{AB}^{T_B}\right)^3$ → classical spin model

Mutual information measures entanglement. Can be read off geometry of network!

Moreover: $I(A:B) + I(A:C) < I(A:BC)$ implies four-partite entanglement.

Generalizes a result of [Smith-Leung] for single stabilizer state.

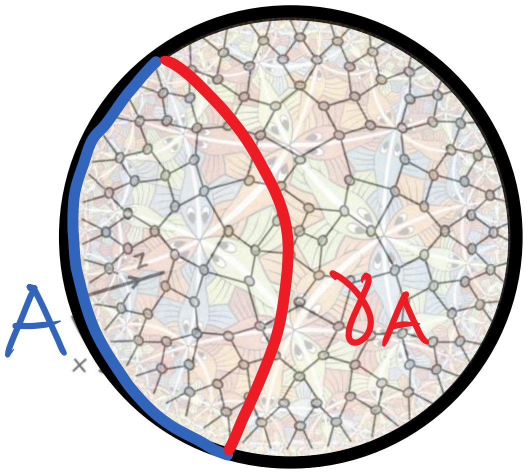
Motivation: Quantum Gravity

[Bekenstein-H.]; [Susskind], [t'Hooft]; [Maldacena]

Holographic principle: All information in a region of space can be represented as a "hologram" living on region's boundary.

$$S_{BH} = \frac{A}{4G_N}$$

AdS/CFT duality: conjectural realization

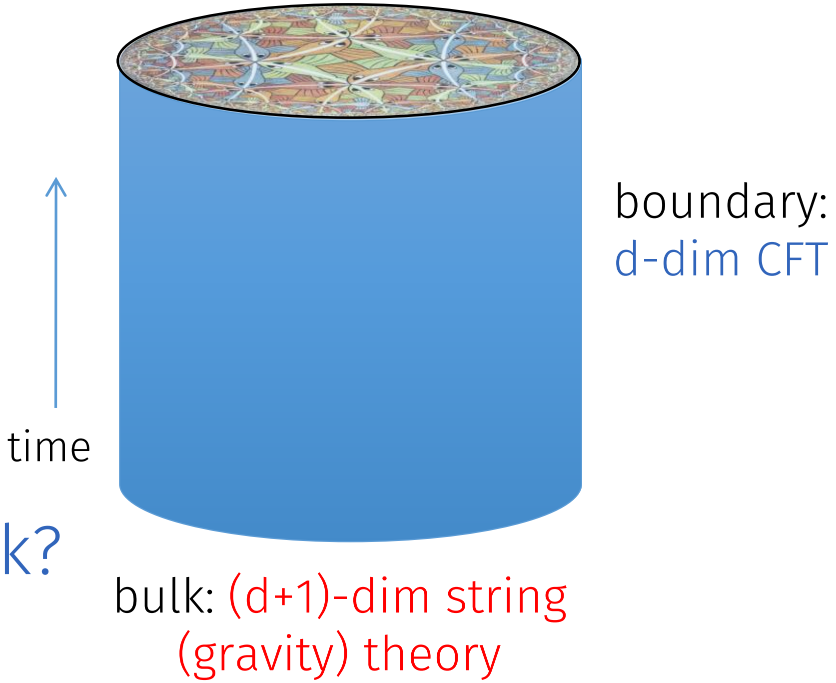


$$S(A) = \frac{1}{4G_N} \min |\gamma_A|$$

[Ryu-Takayanagi]

Space-time as a tensor network?

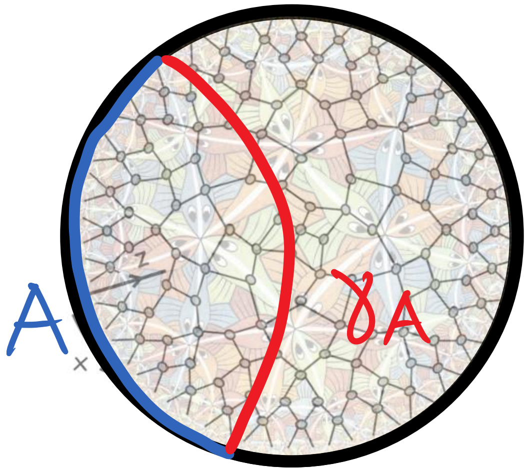
[Swingle]



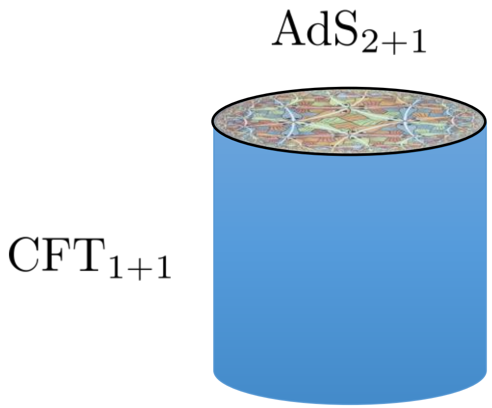
What is the basic mechanism? Fine-tuned or typical phenomenon?

Entanglement entropy in AdS/CFT

Typical behavior of tensor networks with large bond dimensions matches precisely the Ryu-Takayanagi proposal:

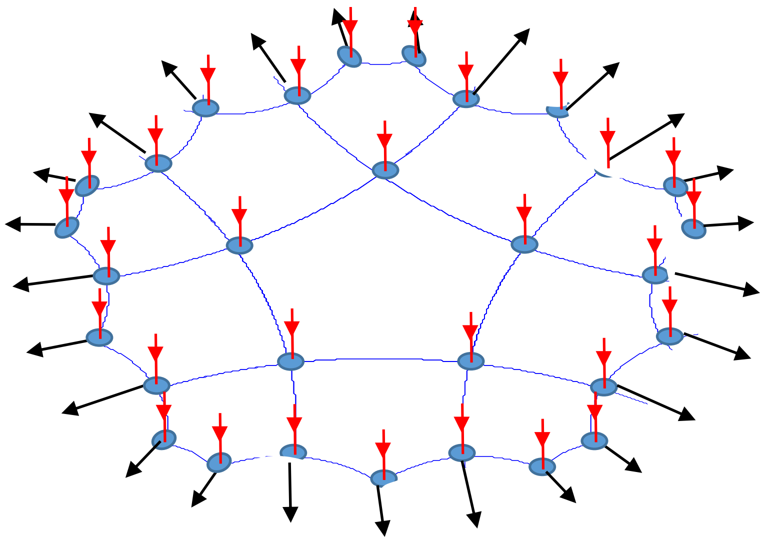


$$S(A) = \frac{1}{4G_N} \min |\gamma_A|$$



<u>Possible interpretation:</u> Fix Planckian d.o.f. of some area-law bulk quantum gravity state to typical values

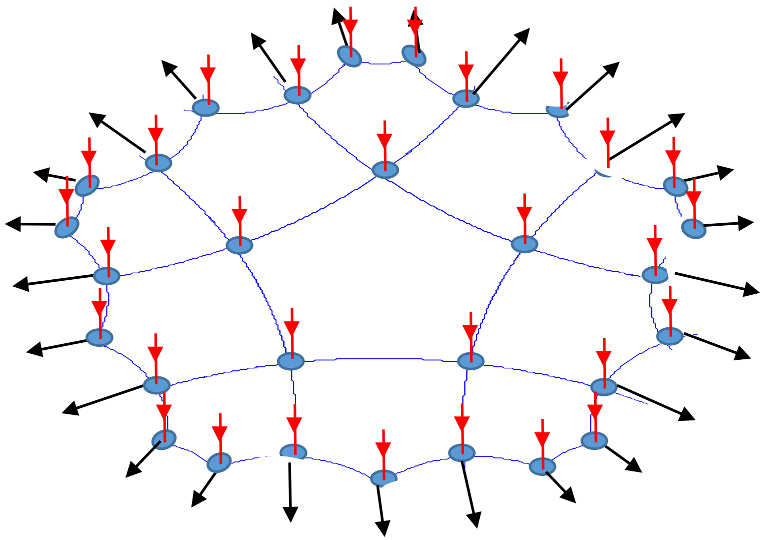
→ merges bulk state into boundary state that satisfies Ryu-Takayanagi formula.



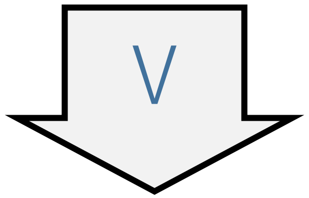
Random tensor networks as holographic mappings

Bulk-boundary mapping from random tensor networks

Tensor network determines “holographic” mapping:



bond dimensions $D_b \ll D$



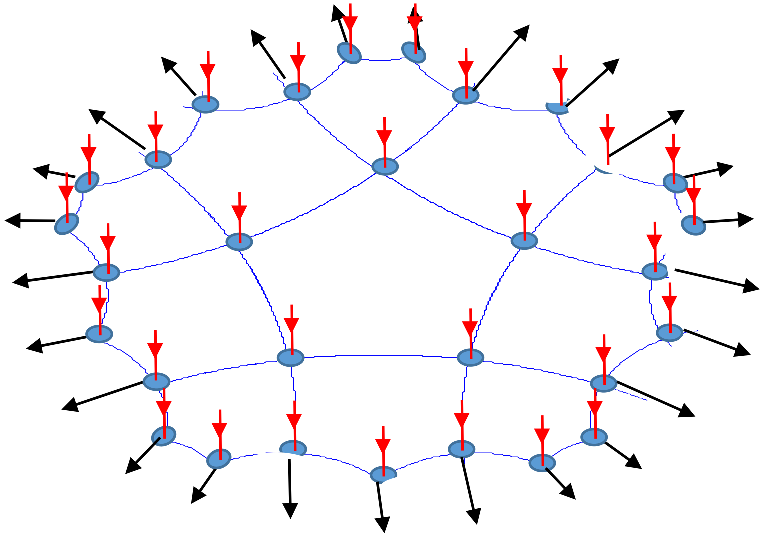
To study properties, highly useful to consider “fictitious” state $|\Psi_{\text{bulk},\text{boundary}}\rangle$.

Bulk-boundary mapping as a quantum code

Holographic mapping is isometry if

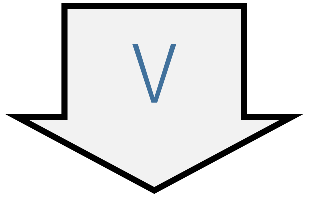
$$S(\text{bulk}) = N_b \log D_b$$

i.e., minimal domain wall cuts off bulk legs



bond dimensions $D_b \ll D$

bulk: logical Hilbert space



boundary: physical Hilbert space

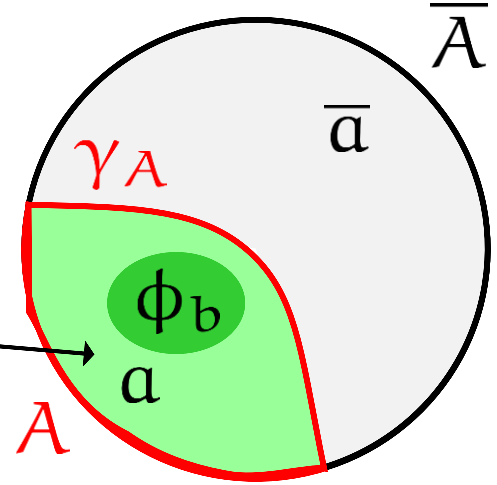
Can faithfully map states and operators: $|\psi_\partial\rangle = V\,|\psi_b\rangle \qquad O_\partial = V \phi_b V^\dagger$

All correlation functions preserved. Entropy formulas hold exactly (w.h.p.) if we use stabilizers.

Locality of the quantum code $\quad O_\partial = V\phi_b V^\dagger$

In general, a logical operator ϕ_b can be realized by various physical operators O_∂.

How local can we choose the latter? When can we implement ϕ_b physically by some O_A?



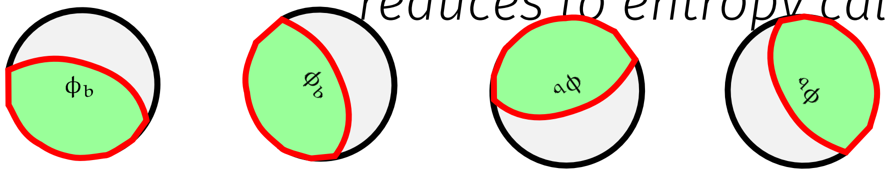
<u>Answer:</u> If supported in "entanglement wedge", the region a enclosed by the minimal cut.

$$I(a : \bar{A}\,\bar{a}) = 0$$

reduces to entropy calculation!

In AdS/CFT: Long conjectured, recently "proved". [Dong-Harlow-Wall]

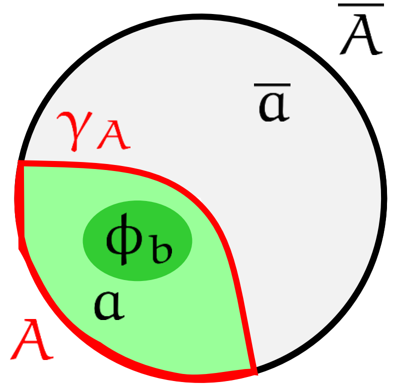
Redundancy in the choice of A. Puzzle?

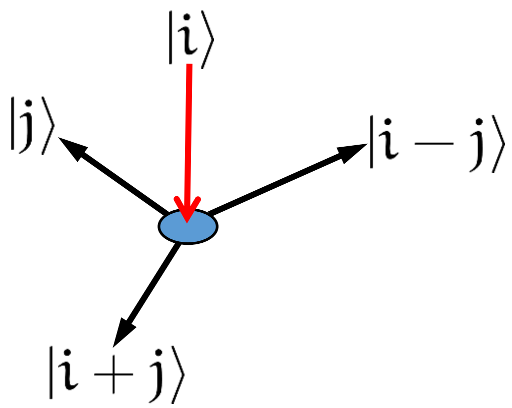


Perfect recovery from A iff a completely decoupled from environment (cf. "no cloning").
Explicit formulas for O_A from recent quantum information results on recovery maps.

Quantum erasure codes

This is a quantum erasure code, such that quantum information deeper in bulk is better protected against erasures. [Almheiri-Dong-Harlow]



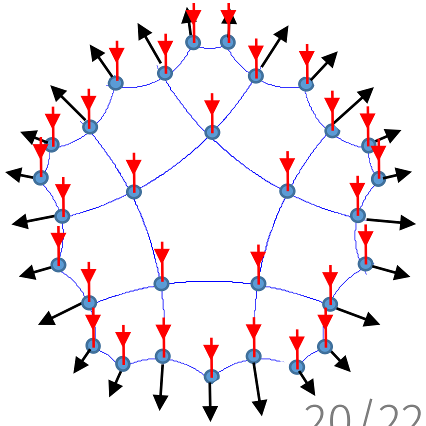


Example: Three-qutrit erasure code [Cleve, Gottesman, Lo]

$$V\phi_b = O_{12}V = O_{23}V = O_{13}V$$

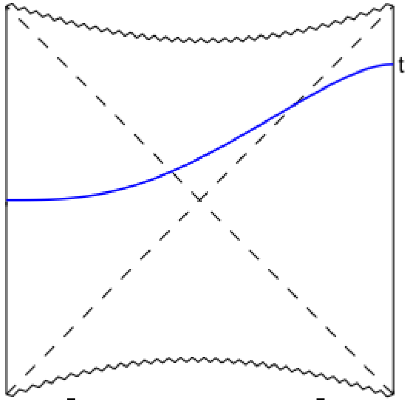
can correct for loss of any single qutrit

→ Networks built from such "perfect" tensors, holographic codes [Pastawski, Yoshida, Harlow, Preskill]



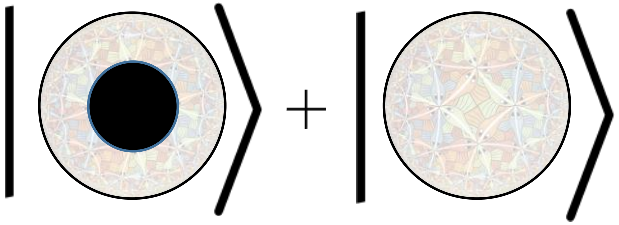
Further ongoing research in tensor networks and q. gravity

Tensor networks discretize space. Gravity is about space-time!



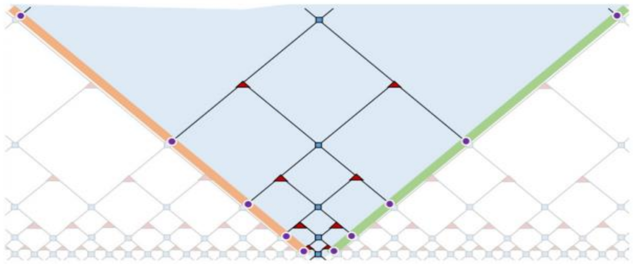
[Hosur et al]

Black hole dynamics believed to be chaotic, scrambling quantum information. Do there exist 'incompressible' circuits?



Superpositions of geometries, causal structures?
Implications on information processing?

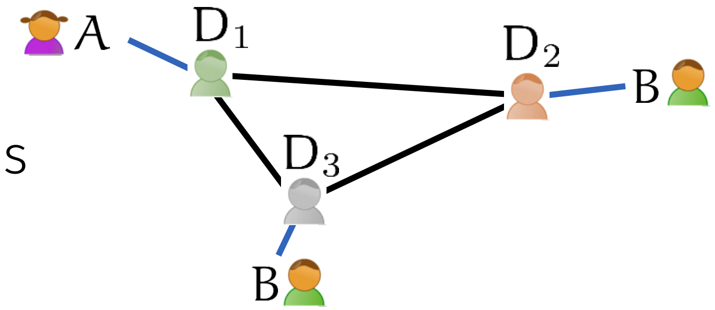
Finding tensor network descriptions of holographic CFT states, also numerically.

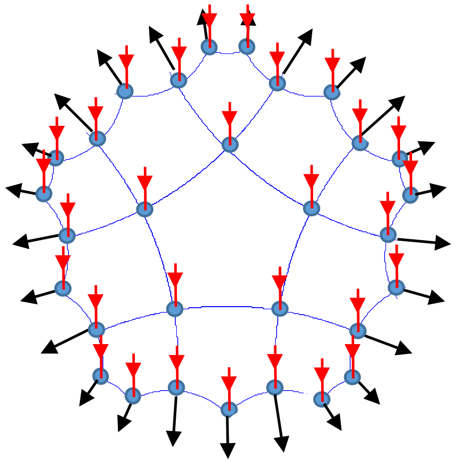

[Czech-Evenbly-...-Sully-Vidal]

Summary

Random tensor networks as a model for studying general mechanism by which quantum information is encoded in tensor networks.

- entanglement structure dictated by geometry
- quantum error correcting codes with interesting locality properties



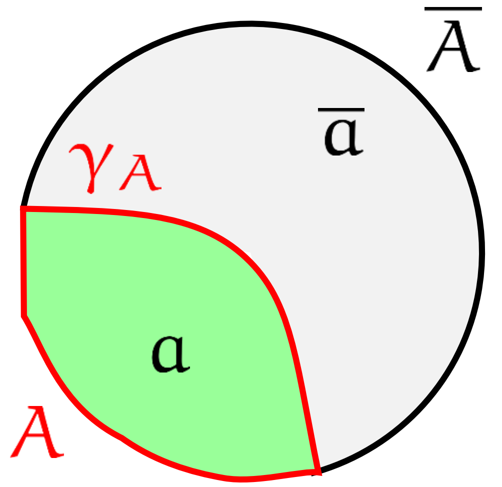


Toy models that reproduce, seek to explain mechanisms behind some of the striking features of the AdS/CFT correspondence.

- ongoing research, many open questions

Thank you for your attention

Locality, relative entropy, recovery



Locality follows from preservation of relative entropies:

$$S(\rho_a \| \sigma_a) = S(\tilde{\rho}_A \| \tilde{\sigma}_A)$$

"logical distinguishability in a = physical distinguishability in A"

where $S(\rho||\sigma) = \rho \log \rho - \rho \log \sigma$. [Dong-Harlow-Wall]

In fact, we can find explicit "recovery map", even in the approximate case:

$$\mathcal{R}[\tilde{\rho}_A] = \rho_a \qquad O_A = \mathcal{R}^\dagger[\phi_a]$$

[Cotler-Hayden-Salton-Swingle-W.]

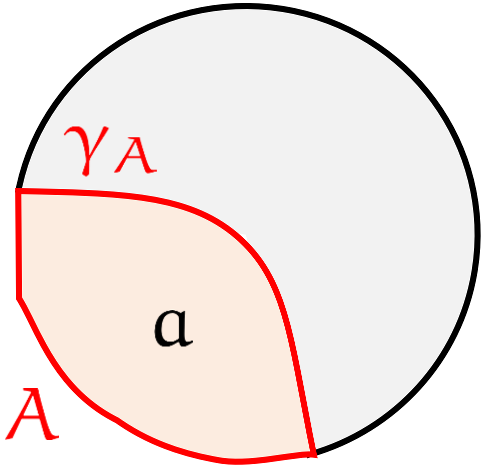
Ingredients: Local bulk-boundary channel $\mathcal{N}[\rho_a] = \mathrm{tr}_{\bar{A}}[V(\rho_a \otimes \tau_{\bar{a}})V^\dagger]$ & recent results on monotonicity of relative entropy by [Junge, Renner et al].

Typical code states $|\psi_\partial\rangle = V|\psi_b\rangle$

What do typical code states (boundary states) look like?

Ising action acquires additional "bulk term". Result:

$$S(A) \simeq \min\{\log D|\gamma_A| + S(a)_{\psi_b}\}$$



Result matches precisely the corrections to the Ryu-Takayanagi formula in AdS/CFT due to entanglement in bulk quantum fields. [Faulkner et al]

Rigorous proof using decoupling technique a la [Dutil-Hayden].

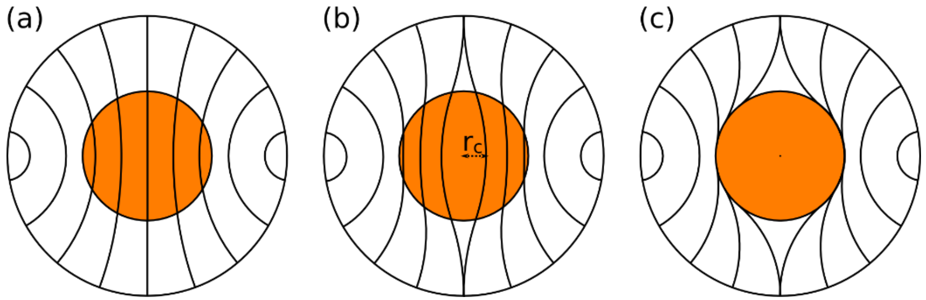
Bulk corrections in AdS/CFT

[Faulkner et al]

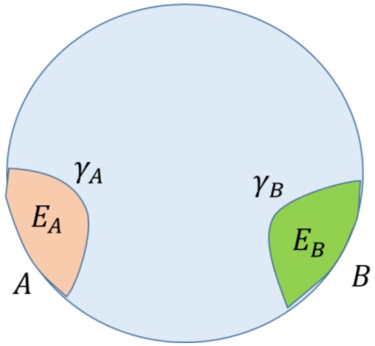
$$S(A) \simeq \min \{\log D|\gamma_A| + S(a)_{\psi_b}\}$$

minimal geodesics get deformed (unless $D_b \ll D$)

entanglement in input state induces correlations in code state



adding massive amounts of bulk entropy: horizons, analog of black hole formation



$$I(A:B) = I(a:b)_{\psi_b}$$

Random tensor networks match precisely the situation in AdS/CFT.