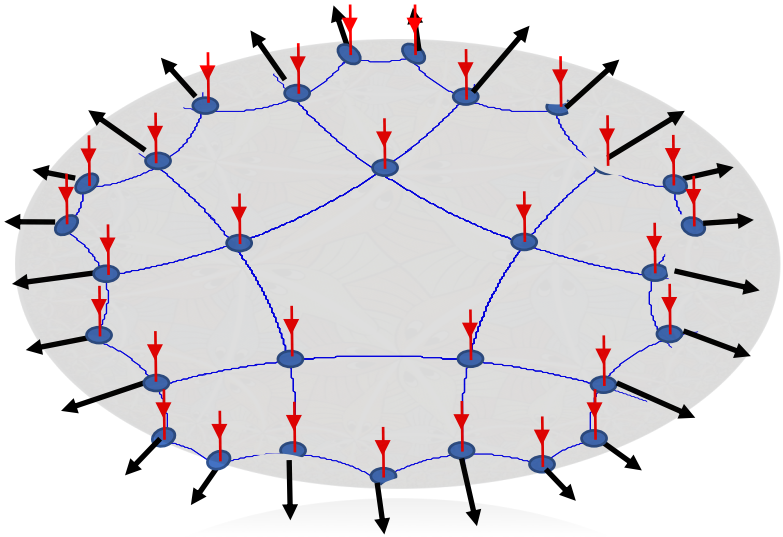
University of Amsterdam

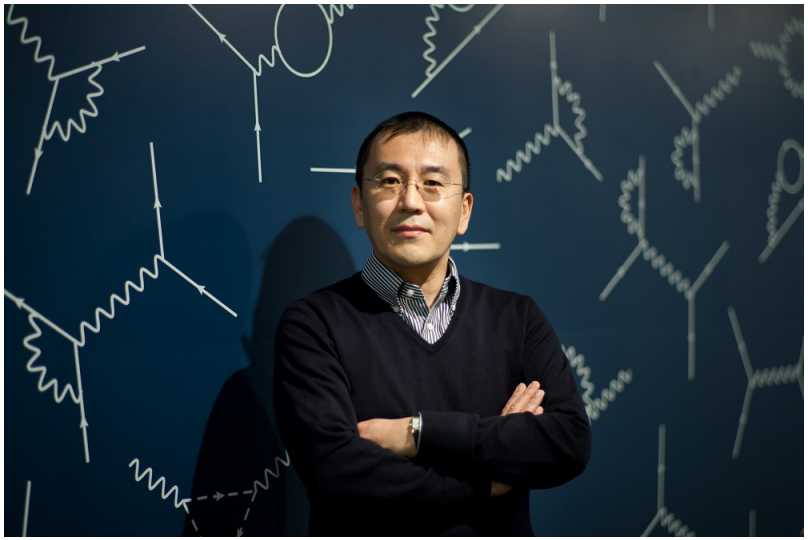
QuSoft
Research Center for Quantum Software

Quantum Information & Holography

Michael Walter



WPC Theoretical Physics Symposium 2018, Hamburg



"The relation between information theoretical concepts in CFT and geometric concepts in AdS has taught us many lessons."

Outline

From Quantum Information to Quantum Fields

Holography and the AdS/CFT Correspondence

Three little pieces:

1. Constraints from Entropy
2. Geometry from Entanglement
3. Dualities as Quantum Codes

Traditionally, want to exploit laws of QM for information processing...

communication

cryptography

networks

algorithms

quantum bits

computation

complexity

Quantum Information

entropy

entanglement

error correction

tensor networks

quantum simulation

...but also provides tools for studying many-body quantum systems!

Many-body quantum states

Quantum states of n qubits have exponentially large description

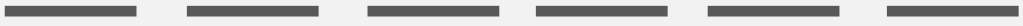
$$|\Psi\rangle = \sum_{i_1,\ldots,i_n} \Psi_{i_1,\ldots,i_n} |i_1,\ldots,i_n\rangle$$

tensor with n indices

In practice: entanglement is local, correlations decay rapidly

→ more efficient descriptions

Key idea: start with entangled pairs…



…and apply local transformations:

e.g. 'cat' state |0…00> + |1…11> from |00> → |0>, |11> → |1>

Tensor networks as a tool

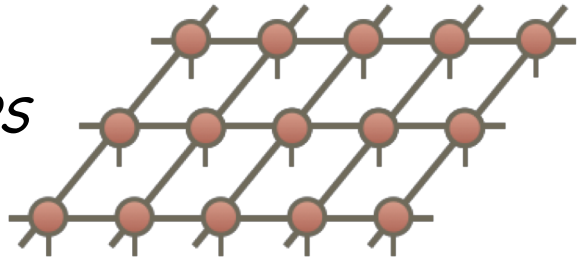
Tensor network: many-body state defined by contracting network of (local) tensors

$$|\Psi\rangle = \sum_{i_1,\ldots,i_n} \Psi_{i_1,\ldots,i_n} |i_1,\ldots,i_n\rangle$$

e.g. *MPS*

White, Fannes-Nachtergaele-Werner, Östlund-Rommer

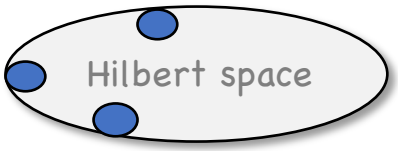
PEPS



Verstraete-Cirac

Numerical tool: efficient variational classes

Can have interpretation as quantum circuits

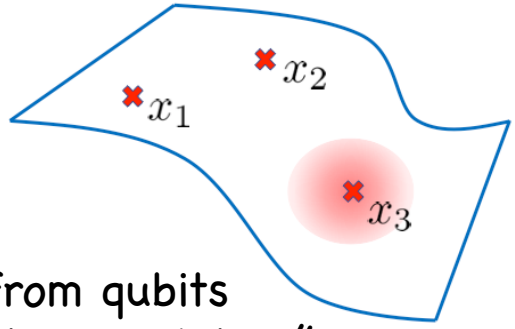


Powerful theoretical formalism that provides "dual" descriptions of complex phenomena → quantum phases, topological order, ...

Quantum information & field theory

Do quantum information tools apply to quantum field theory?

Continuum as a challenge: Notions such as subsystems, entropy, approximation, circuits become more subtle!



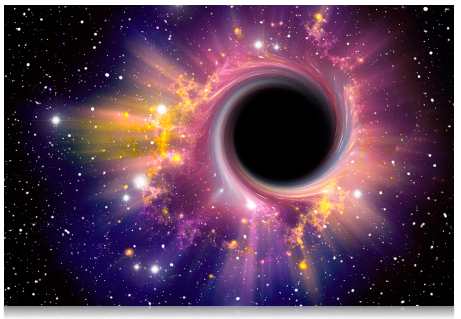
"from qubits to correlators"

→ talk by Ignacio Cirac

Theoretical insights: c-theorem from subadditivity, Bekenstein bound from relative entropy, renormalization as QEC...

Quantum computers will be useful for simulating quantum physics. Can we simulate QFTs, or even black holes in quantum gravity...?

Black holes and quantum information



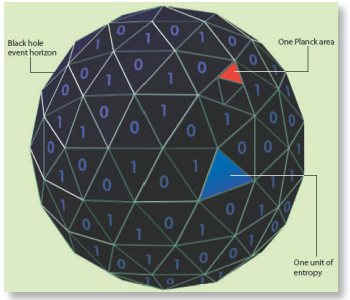
Black holes have a thermodynamic temperature and entropy. This entropy is proportional to the area of the event horizon:

$$S_{BH} = \frac{\text{Area}}{4G}$$

Bekenstein
Hawking

Surprising! Further puzzles arise when we try to quantize: Hawking radiation, information paradox(es), ...

A theory of quantum gravity ought to give microscopic explanations.

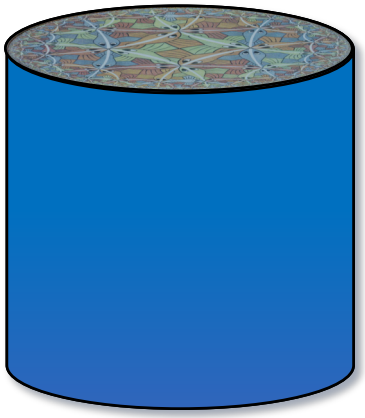


Holographic principle and practice

Holographic principle: Can all information in a region of space be represented as "hologram" living on boundary?

Susskind
't Hooft

AdS/CFT duality: Realization in Anti-de Sitter space Maldacena



Not our universe!

But controlled setup to study quantum gravity; including black holes, wormholes, ...

What can we learn by applying the QI toolkit?

1. Constraints from Entropy

Quantum entropy

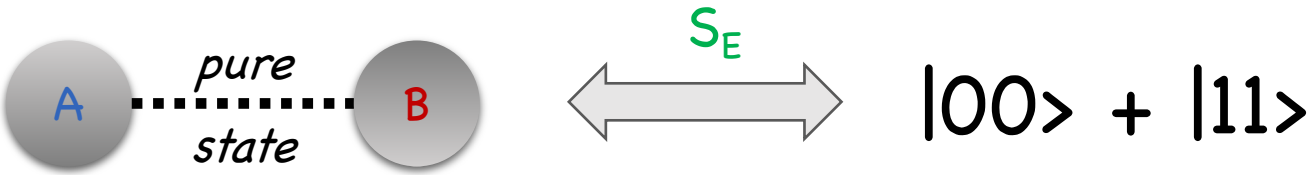
Entropy is a central quantity in quantum information theory:

$$S(A) = -\mathrm{tr}[\rho_A \log \rho_A]$$

quantum system A described by density matrix ρ_A

Many interpretations and uses in optimal rates & capacities:

Entanglement entropy: $S_E = S(A) = S(B)$

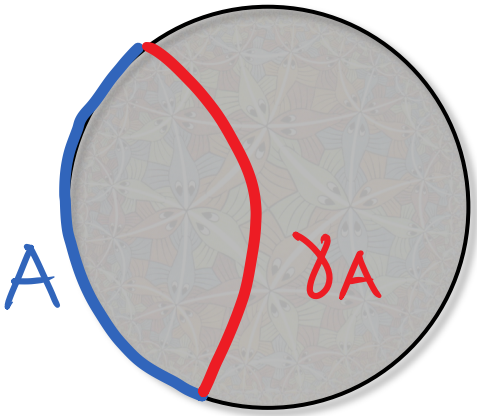


Mutual information: $I(A{:}B) = S(A) + S(B) - S(AB)$

bounds correlations

Entropy in holography

Boundary entropies are given by areas of bulk minimal surfaces:



time slice

$$S(A) = \frac{|\gamma_A|}{4G} + \ldots$$

Ryu-Takayanagi (RT)

Implications for CFT state? Conversely, can we use known properties of entropy to constrain the gravity side?

It is easy to verify known entropy inequalities such as the strong subadditivity property. However, we can prove "too much"...

Holographic entropy laws

Ryu-Takayanagi formula satisfies non-standard entropy inequalities.
These constrain theories of quantum gravity!

"Monogamy" inequality:

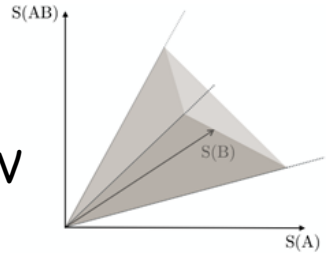
$$I(A{:}B) + I(A{:}C) \leq I(A{:}BC)$$

Hayden-Headrick-Maloney

$I(A{:}B) = S(A)+S(B)-S(AB)$
recall mutual information

does not even hold for all probability distributions.
reason: classical correlations are not monogamous.

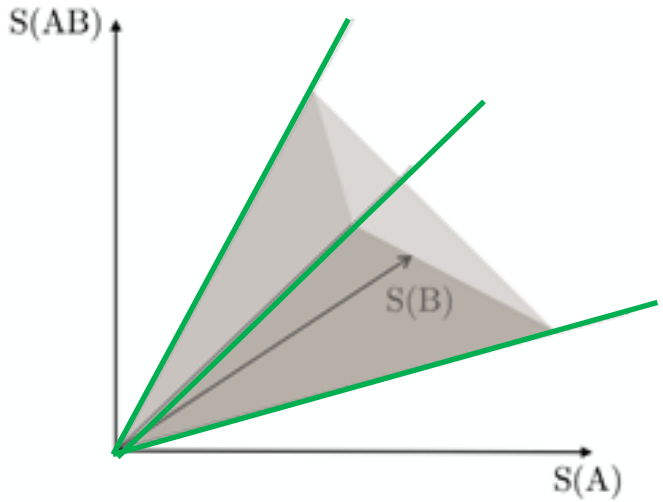
Infinitely many holographic entropy inequalities,
but can be organized systematically. Bao-...-Ooguri-W



Holographic entropy laws

Holographic entropy cones

Bao-...-Ooguri-W



example:
2 subsystems

faces: entropy inequalities such as S(A) + S(B) >= S(AB)

rays: extremal geometries. can we identify these with microscopic building blocks of holographic states?

will come back to this question later...

Constraints from entropy inequalities

Can also go the other way and exploit known entropy inequalities to derive gravitational constraints. E.g., using relative entropy:

$$S(\rho\|\sigma) = \mathrm{tr}[\rho \log \rho] - \mathrm{tr}[\rho \log \sigma] \geq 0$$

Perturb around vacuum state:

1st order: linearized Einstein equations — Faulkner et al

2nd order: positive energy inequalities — Lin et al, Lashkari et al

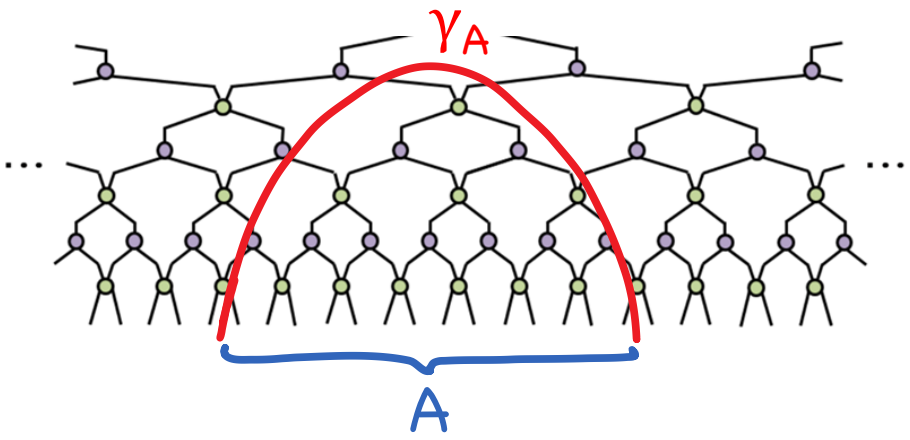
e.g. $$\int T_{00} \sqrt{g} \geq 0$$

Much more to be said about holographic entropies (monotonicity of relative entropy, Freedman-Headrick bit threads, ...)

2. Geometry from Entanglement

Entropy in tensor networks

Entanglement entropy in tensor networks satisfies "area law":



$$S(A) \leq N\,|\gamma_A|$$

N qubits/bond

γ_A = minimal cut

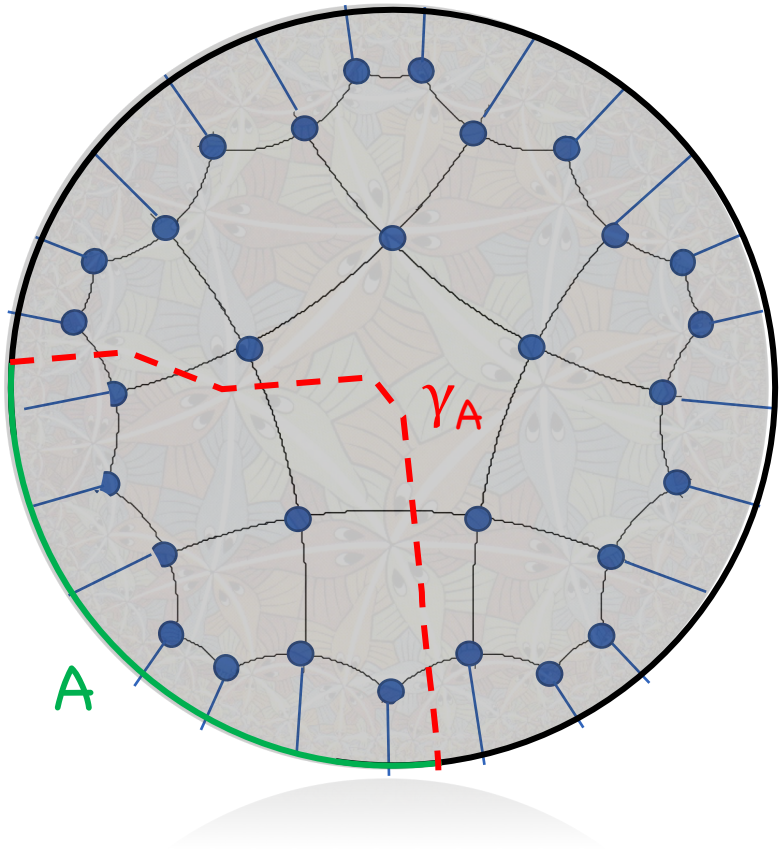
Looks like RT formula! In general, the bound is not saturated...

Tantalizing: Picture shows Vidal's MERA tensor network. Swingle

Used for critical theories, it looks like a time slice of AdS!

Holography from tensor networks

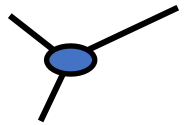
Want "exactly solvable" toy models of holographic duality:



Harlow et al, Hayden-...-W

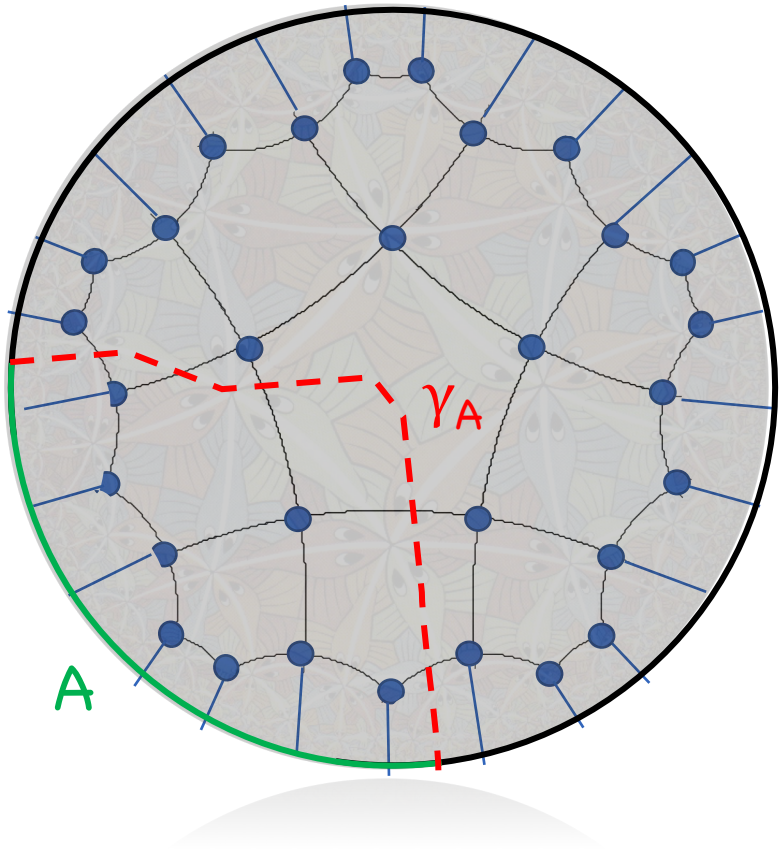
Approach: Define boundary state via tensor network in bulk

simple bulk tensors, e.g. random tensors



Holography from tensor networks

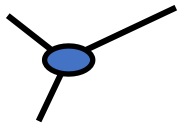
Want "exactly solvable" toy models of holographic duality:



Harlow et al, Hayden-...-W

Approach: Define boundary state via tensor network in bulk

simple bulk tensors, e.g. random tensors



For large N, emergent Ryu-Takayanagi law!

$$S(A) \simeq N\,|\gamma_A|$$

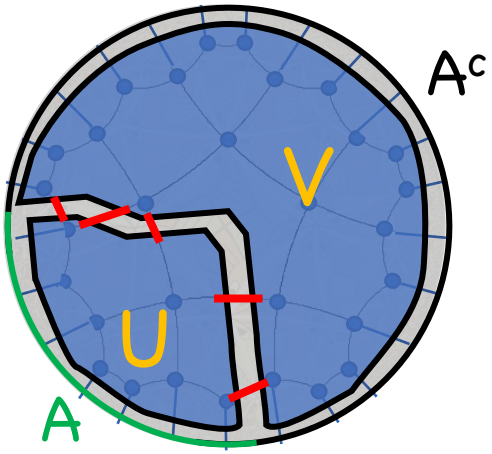
✓

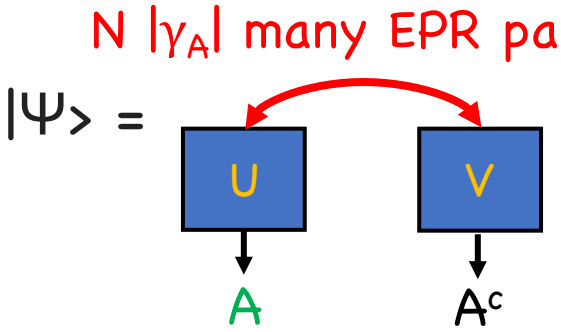
Mostly works in any geometry. By now, many variations known.

Three interpretations

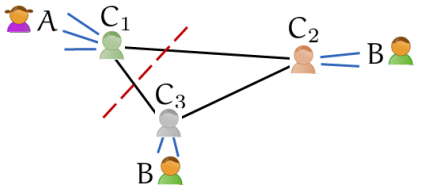
Harlow et al,
Hayden-…-W

1. Random tensors ≈ unitary gates in any direction ("perfect tensors")



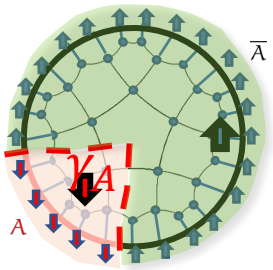


2. Entanglement distillation protocol



3. Disorder average → ferromagnetic spin model

large N → low T

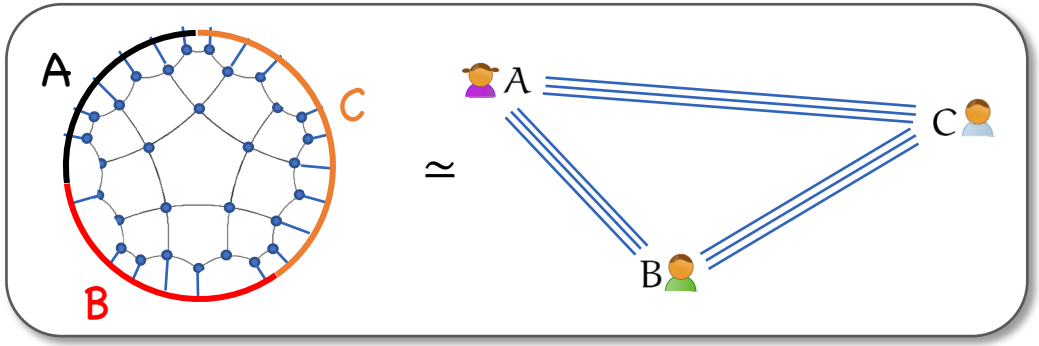


Interpretation of random tensors? Typicality of Planckian bulk degrees?

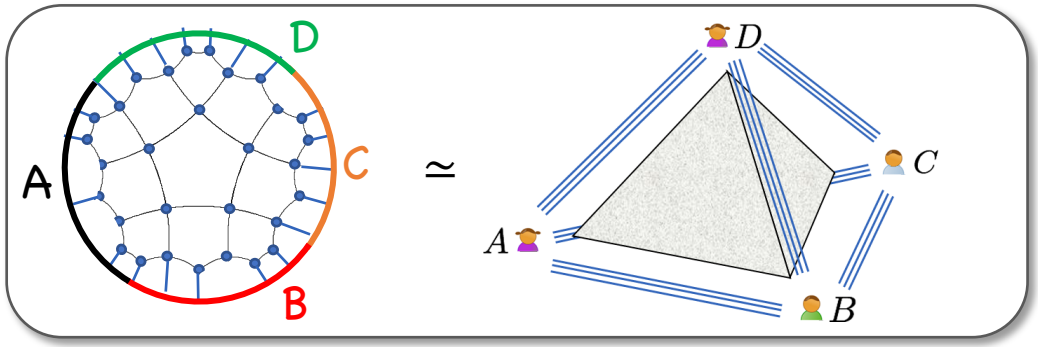
Dissecting holographic states

Nezami-W

Inspired by entropy cones, decompose states into building blocks:



state dominated by EPR entanglement



"perfect tensor"-type entanglement emerges

- ✓ agrees with entropy cone
- ✓ new proof of monogamy
- ✓ bit threads are not enough?

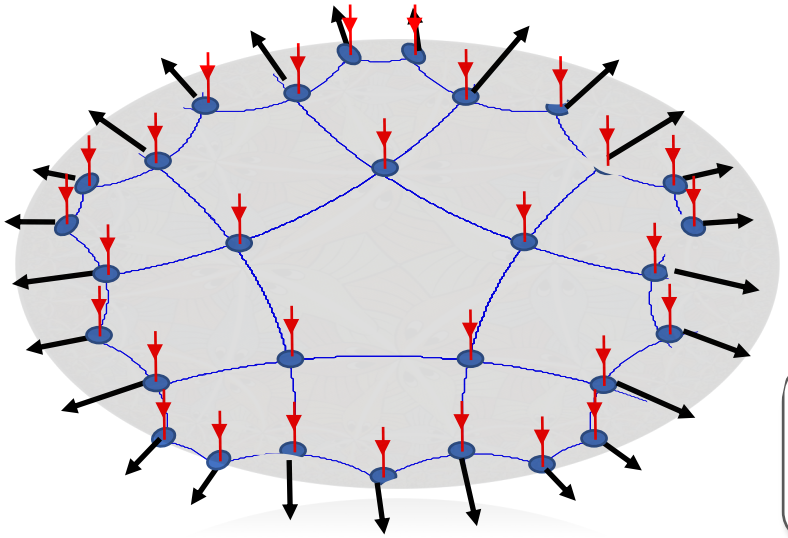
Does this also hold in AdS/CFT? Need new tools! Cui-...-W

3. Dualities as Quantum Codes

Holographic codes

AdS/CFT is duality between two theories…
a whole "dictionary", mapping states & observables

Approach: Define bulk-boundary mapping via tensor network



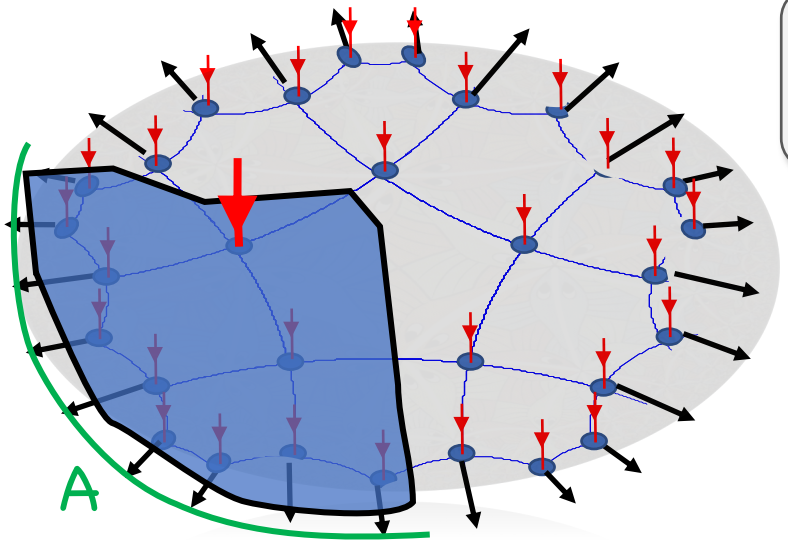
red legs: bulk degrees
black legs: boundary degrees

"logical" bulk states are encoded in "physical" boundary Hilbert space

toy model of how bulk quantum fields get encoded in boundary CFT

Locality & error correction

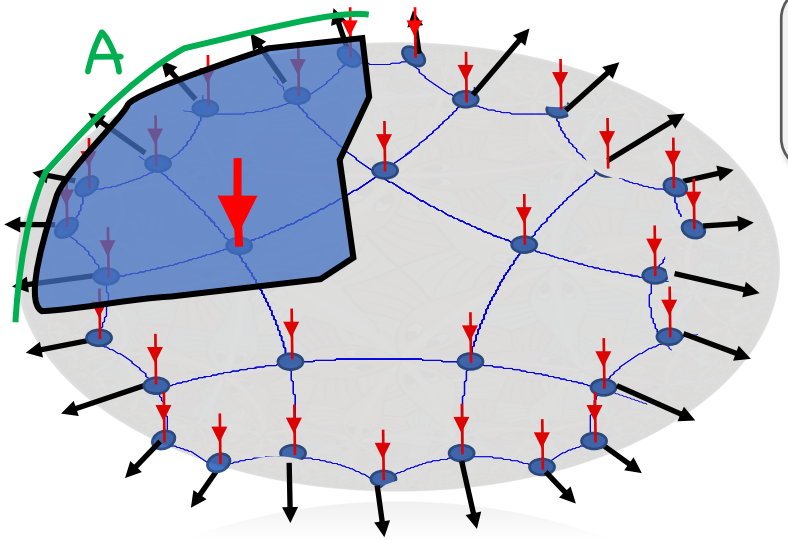
When can we reconstruct bulk qubit from boundary system?



Answer: if in "entanglement wedge", region enclosed by minimal cut

Locality & error correction

When can we reconstruct bulk qubit from boundary system?



Answer: if in "entanglement wedge", region enclosed by minimal cut

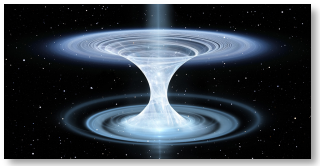
This region is not unique. Paradox?

Almheiri-Dong-Harlow

No. Redundancy is feature of q. error correcting code!

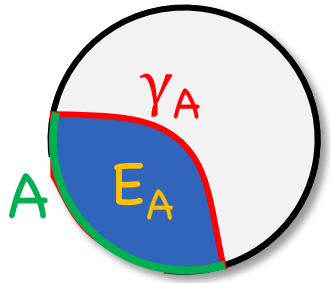
Q. information deep in bulk is better protected. Holographic codes are macroscopic erasure codes built from microscopic ones (perfect tensors)

ER = EPR?



Tensor network models reproduce several quantum information features of AdS/CFT correspondence:

✓ error correction in entanglement wedge



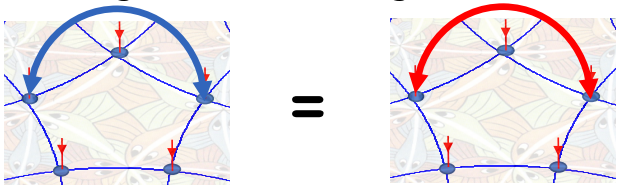
Dong-Harlow-Wall

Faulkner et al

✓ bulk corrections to entanglement entropy:

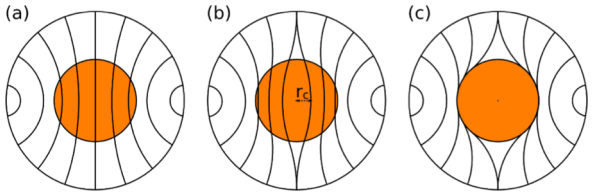
$$S(A) \simeq \min \{ N\,|\gamma_A| + S(E_A) \}$$

entanglement vs geometry:



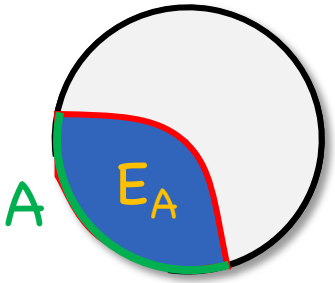
Maldacena-Susskind, Verlinde, ...

adding too many states "breaks" code and creates entanglement shadow (≈ horizon)



cf. BH microstates

Decoding the hologram (using error correction)



Original proof of "entanglement wedge" reconstruction property was nonconstructive & nonrobust

based on exact decoupling duality in q. information

How to find boundary reconstruction of local bulk operator?

Banks et al, Hamilton et al, Kabat et al, Heemskerk et al, Lin et al, Faulkner-Lewkowycz, ...

Understood in special cases. But not when operator behind horizon! Similarly, how to decode Hawking radiation?

State dependence? How large can "code subspace" be?

Recent progress in theory of quantum error correction may lead to more explicit formulas and decoding protocols.

Cotler-...-W, Kitaev-Yoshida, Hayden-Penington

The road ahead

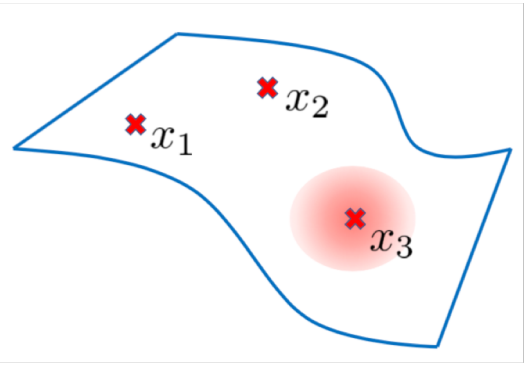
Tensor networks discretize space, but gravity is about space-time: *dynamics, backreaction, causal structure?*

Q. information vs geometry: holography in flat space & de Sitter? superpositions of geometries?

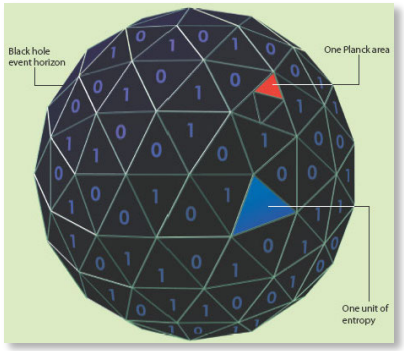
Practical diagnostics for entanglement and correlations

What makes a CFT gravitational?

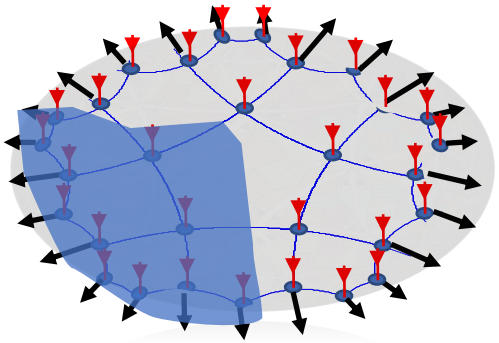
Continuum limits of states and circuits



Summary



Holography predicts remarkable connection between geometry and entanglement



Quantum information offers new tools, models, mechanisms from tensor networks to QEC

Ongoing research to exploit connections

Motivation ranges from trying to understand the emergence of space-time from quantum mechanics to learning how dualities can help simulate complex quantum systems on (quantum) computer…

Thank you for your attention!