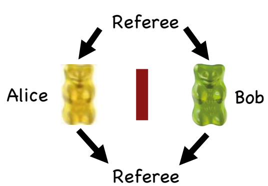
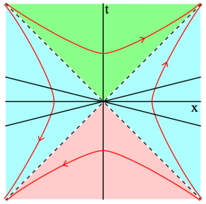
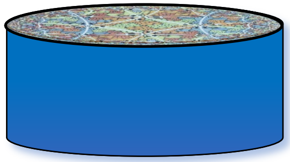
Trading Space for Time in Nonlocal Games

Michael Walter







Quantum Extreme Universe Workshop, Okinawa, Oct 2024

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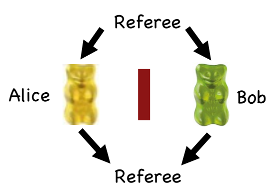
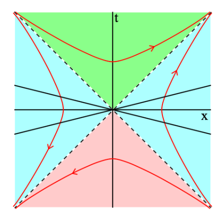
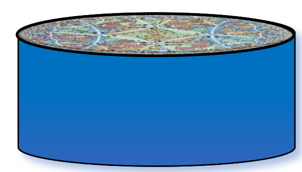
Trading Space for Time in Nonlocal Games

or: Playing Games with Locality

Michael Walter







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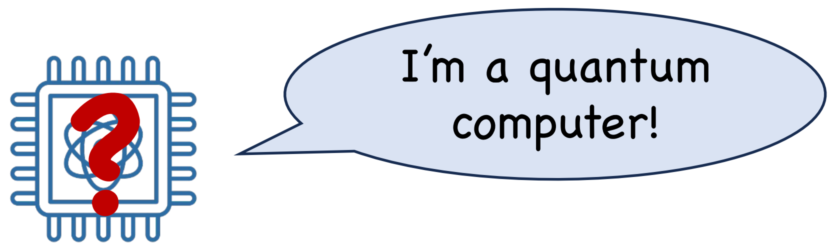
Can one trust a quantum computer?



Can a classical "verifier" convince themselves that they are indeed interacting with a quantum computer?

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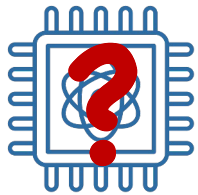

I’m a quantum computer!

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Idea: Violate a Bell inequality.

[TU Delft]

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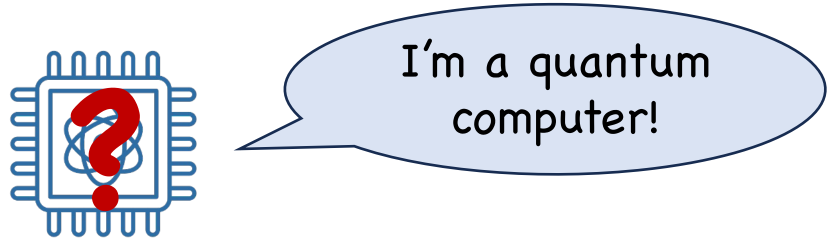

Can a classical "verifier" convince themselves that they are indeed interacting with a quantum computer?

[TU Delft]

Idea: Violate a Bell inequality.

Easy to verify. Remarkably, can be extended to verify arbitrary quantum computation!

[Reichardt et al, ..., Grilo]

☹ need two devices
☹ need to ensure they are spacelike

Can one trust a quantum computer?

Can a classical “verifier” interacting with single device do the job?

Possibly! Any quantum computation can be simulated classically, but in general only *inefficiently* *(or so we believe)...*

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☺ easy to verify
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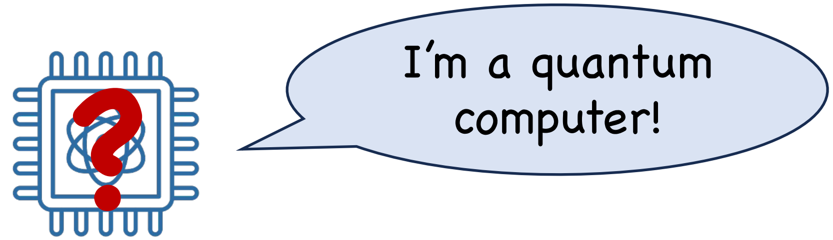


Possibly! Any quantum computation can be simulated classically, but in general only *inefficiently* *(or so we believe)...*

Idea: Ask it to factor a number.

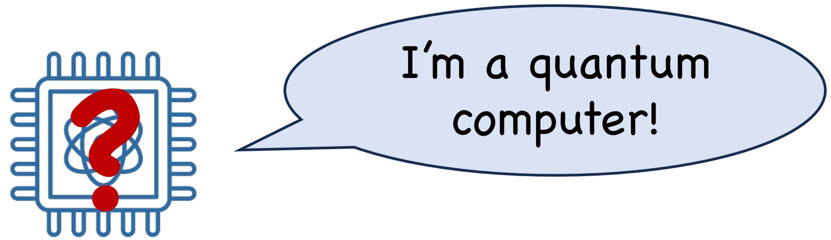
☺ easy to verify
☹ not universal – only certifies that we can factor

Better idea: Ask it to solve universal (BQP-complete) problem.

Can be made to work. Breakthrough gave first classical verification protocol for single device *under computational assumptions*. [Mahadev]

namely that device is efficient & some computational problem is hard

How can one trust a quantum computer?



Nonlocal Approach

- need two <u>spacelike</u> devices
- no assumption on inner workings

Computational Approach

- a single device is enough
- need to assume device is <u>efficient</u> (polynomial-time)

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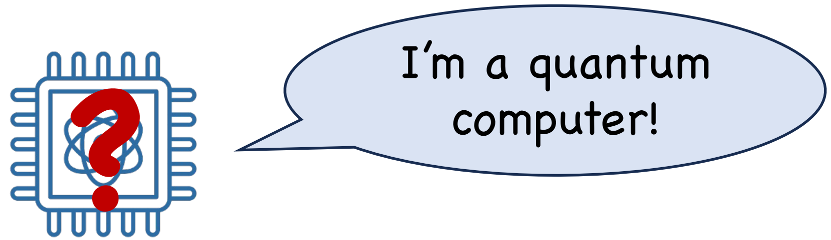

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???

Computational Approach

- a single device is enough
- need to assume device is <u>efficient</u> (polynomial-time)

Question: Is there a systematic link between these two worlds?

Cf. how can locality emerge for bounded complexity observers?

Nonlocal games

[Bell, Clauser-Horne
-Shimony-Holt, ...]

Two non-communicating players play against a referee:

Referee

Alice   Bob

Nonlocal games

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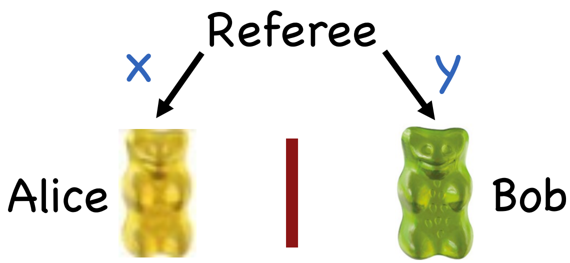

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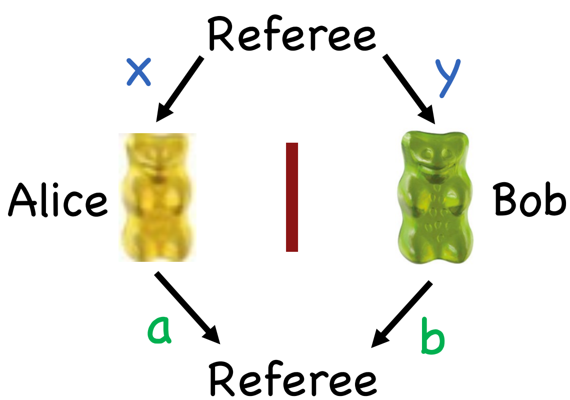

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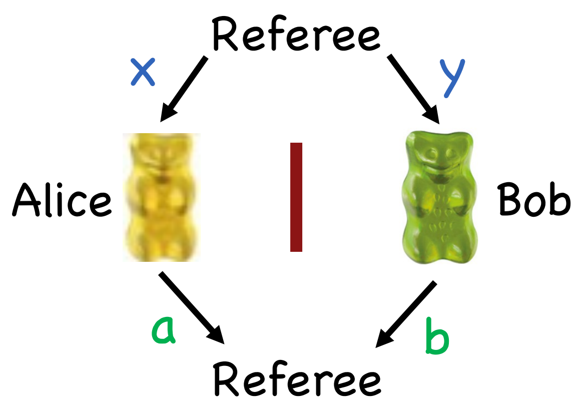

Winning Condition (**CHSH Game**)

x	y	a + b
0	0	even
0	1	even
1	0	even
1	1	odd

Question: Can classical players win this game?

Nonlocal games

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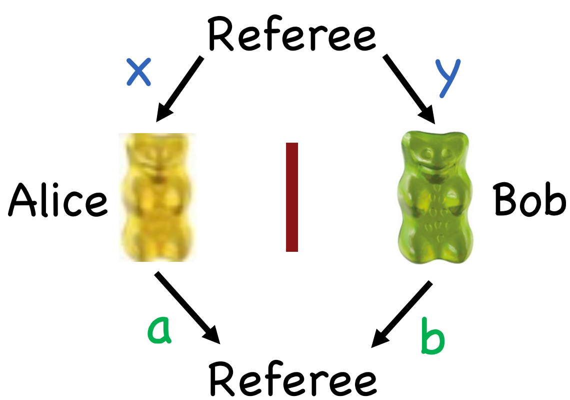

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x	y	$a(x) + b(y)$
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Are there suitable "answer functions" $a(x)$, $b(y)$? If so, then...

$$\big(a(0) + b(0)\big) + \big(a(0) + b(1)\big) + \big(a(1) + b(0)\big) + \big(a(1) + b(1)\big)$$

...would be **odd**. But each answer appears **twice**. **Contradiction!**

Nonlocal games

[Bell, Clauser-Horne -Shimony-Holt, ...]

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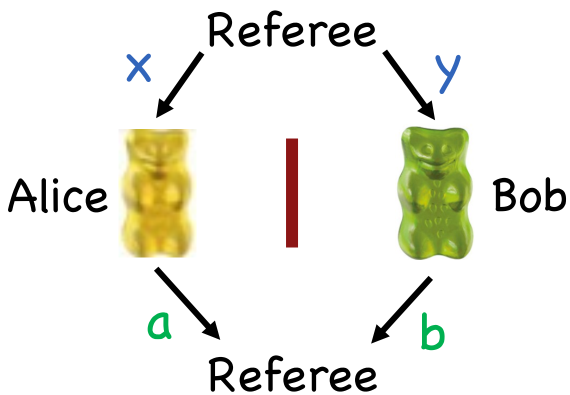

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There is no "classical" way to win CHSH game: $p_{\text{win}}^{\text{classical}} \leq \frac{3}{4}$

This is a Bell inequality!

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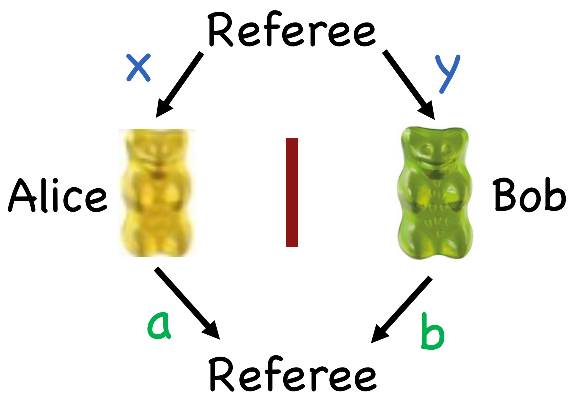

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[TU Delft]

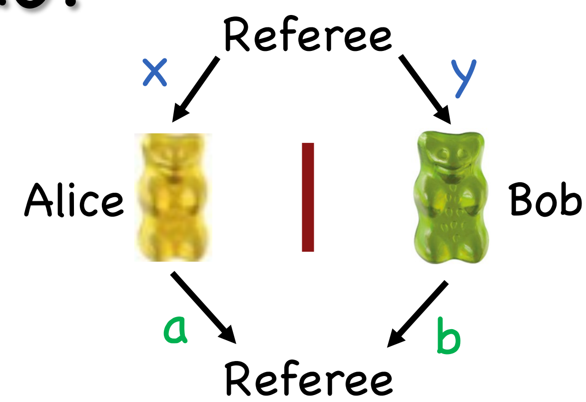
If the players share quantum entanglement they can do better!

How well can quantum players do?

If they share EPR pair $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and use complementary measurements, can achieve:

$$p_{\text{win}}^{\text{quantum}} \approx \mathbf{85\%}$$

and this is optimal [Tsirelson]



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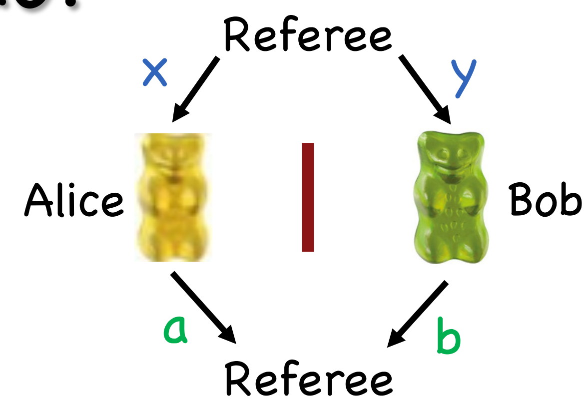

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"Operational" characterization of entanglement!

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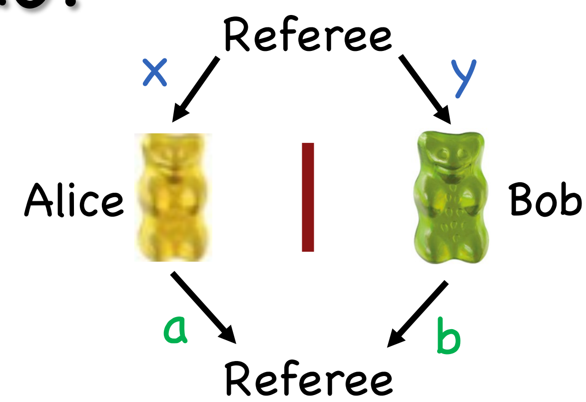

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"Operational" characterization of entanglement!

Classical verifier can verify & control untrusted pair of quantum devices [Reichard-Unger-Vazirani, ...]

→ *device-independent cryptography*

Reason: Hidden symmetry! Roughly, ϵ-optimal strategy ⇔ ϵ-representation of G = <X,Z>, and there is a nearby exact representation [Gowers-Hatami] ☺

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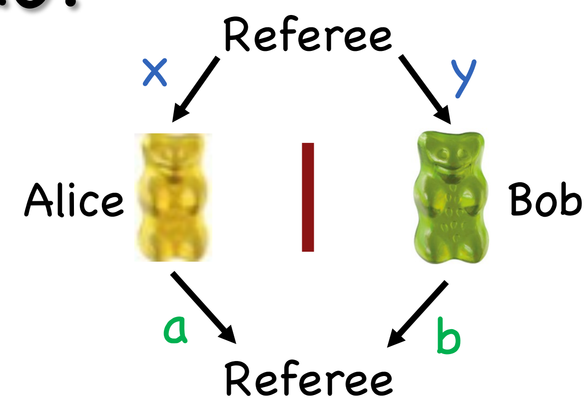

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Classical verifier can verify & control untrusted pair of quantum devices

So long they can't communicate (are spacelike)!

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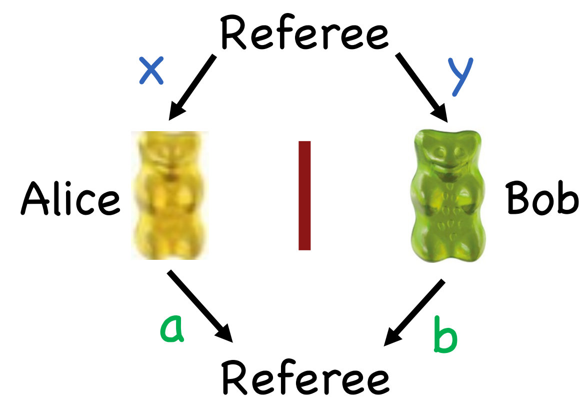
Nonlocal games and their values



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> The optimal winning probability for some class of strategies is called a **"value"**:

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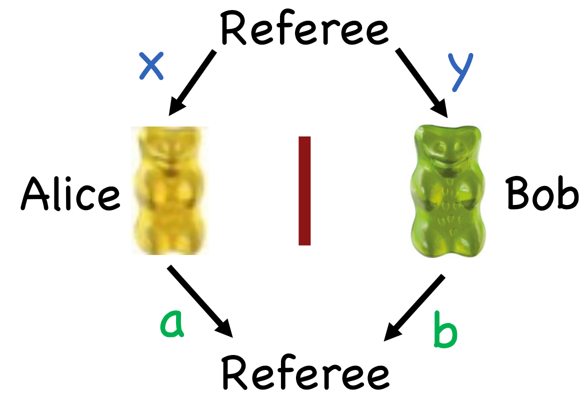

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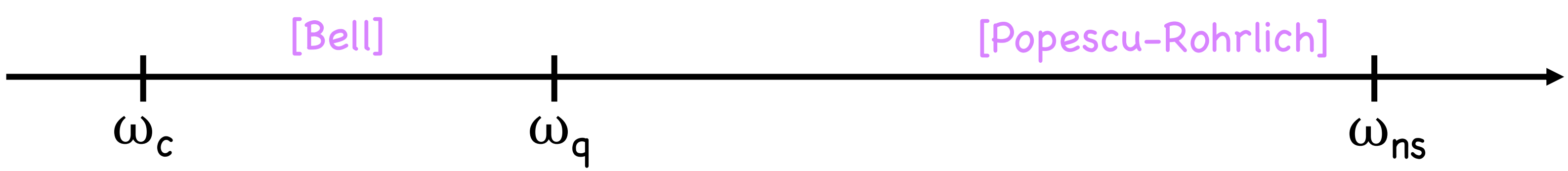

Classical value

Quantum value

Non-signaling value

$p(a|xy) = p(a|x)$
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Nonlocal games and their values

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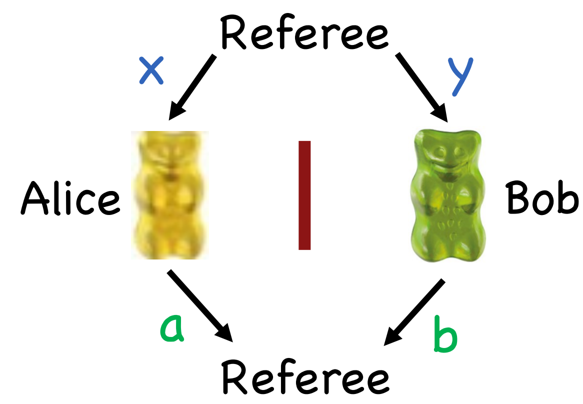

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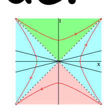
[Bell] [MIP*=RE] [Popescu-Rohrlich]

ω_c ω_q ω_{qc} ω_{ns}

Classical value

Quantum value
$H_A \otimes H_B$

Quantum commuting operator value:
$[A,B] = 0$ 

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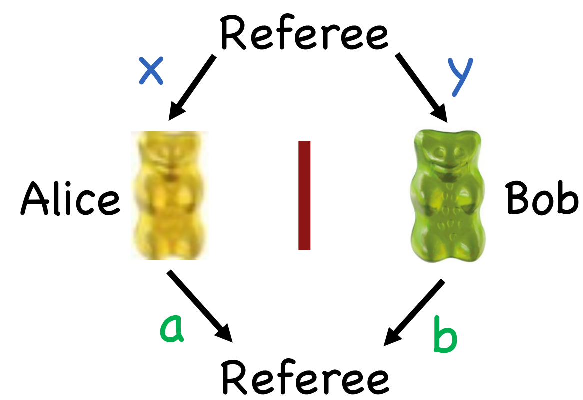

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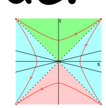
$[A,B] = 0$ 

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In general **all distinct**!

Crucially, <u>no</u> assumption about the player's efficiency!

Trading space for (polynomial) time

Question: Can we get rid of spacelike separation and play a nonlocal game with a **single** efficient player (“prover”)?

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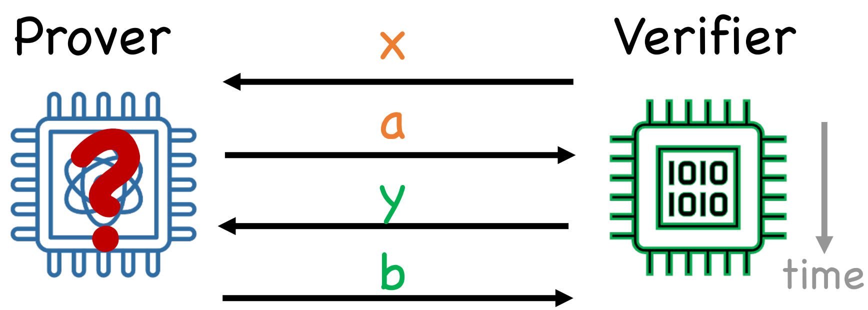
Naïve attempt: Just play sequentially.



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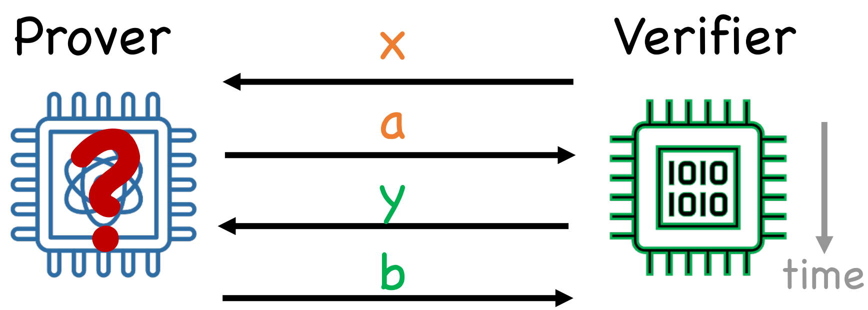

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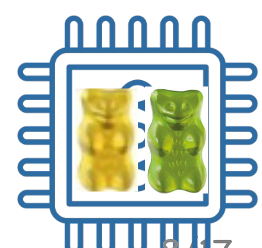
Idea: Use cryptography to force prover to "simulate" two spacelike players **(as long as they are unable to break the cryptography)**!

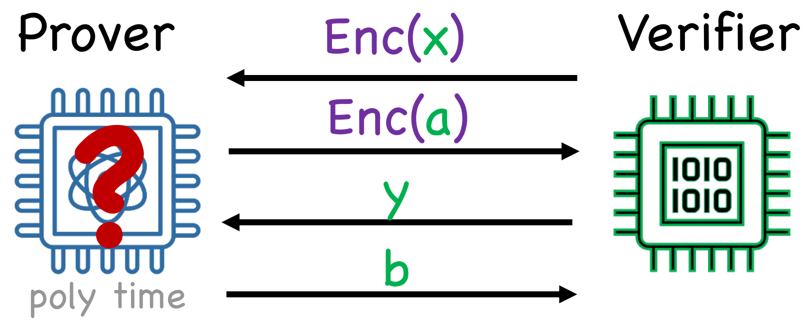


long history in crypto

Trading space for time – the KLVY way

Kalai-Lombardi-Vaikuntanathan-Yang:

Encrypt Alice's question x, while sending Bob's question y in plain.



Intuition: Since prover does not know secret key, encrypted messages cannot usefully be "combined" with plain ones (as if they were spacelike)?

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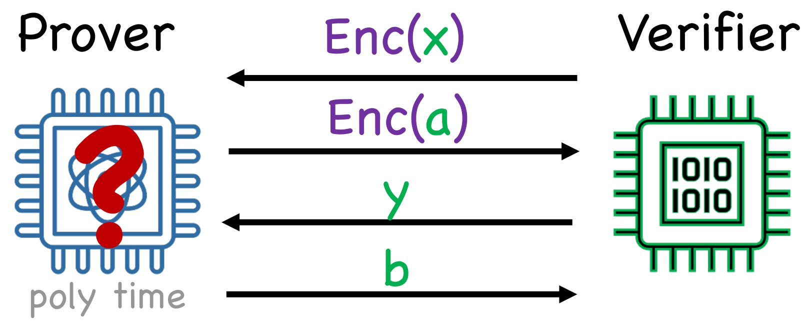

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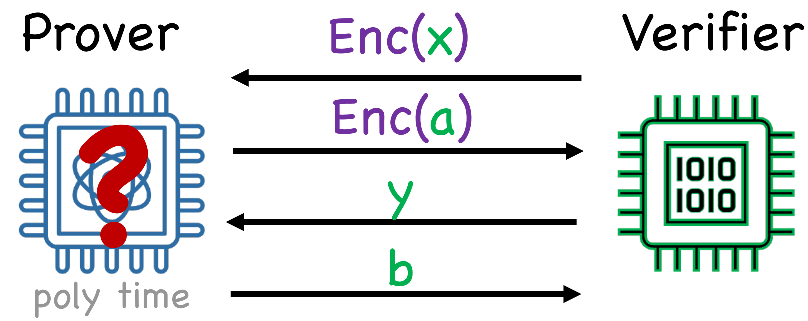

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Solution: Use "homomorphic" encryption scheme!

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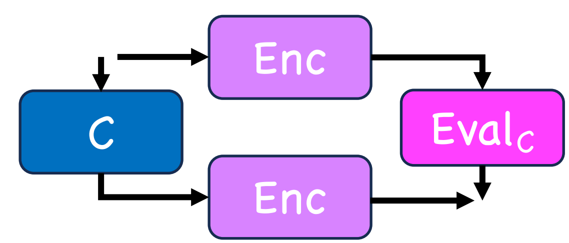

for circuit C

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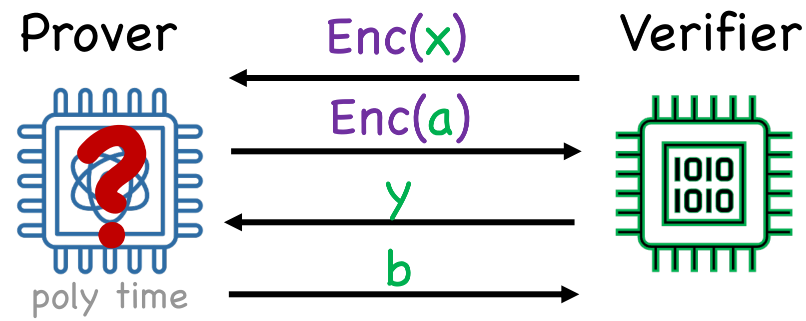

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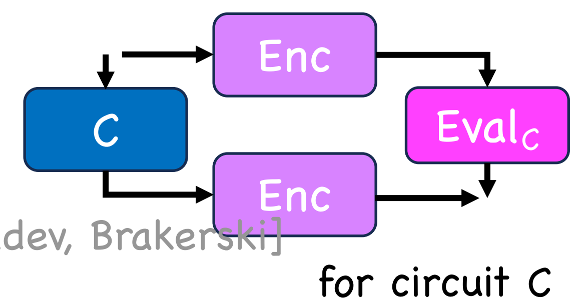
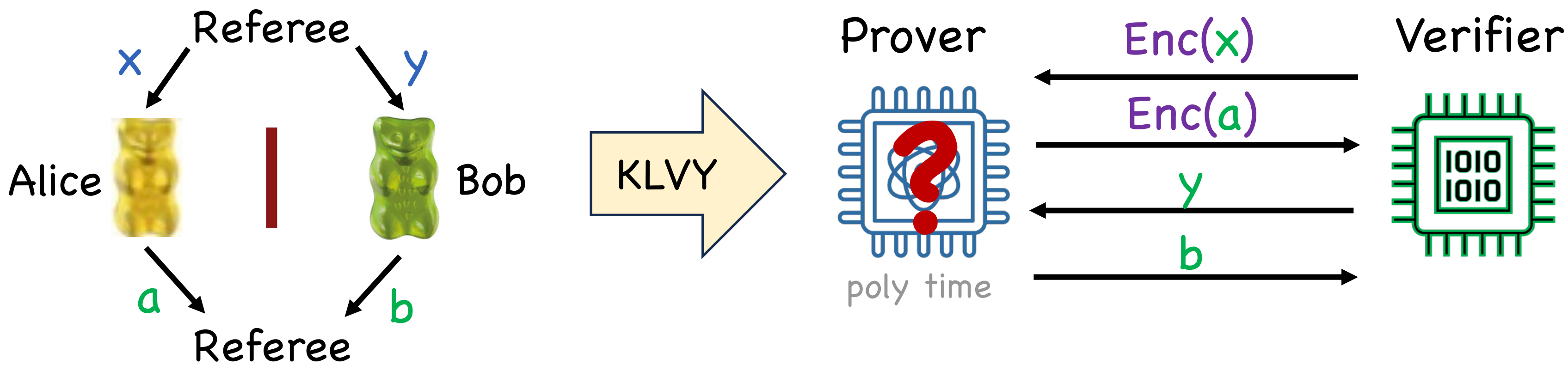


These exist under computational assumptions. [Mahadev, Brakerski]

→ General "compiler" that applies to any nonlocal game ☺

Trading space for time – the KLVY way

Given any nonlocal game, can "compile" into a single-prover protocol:



Key question: What properties of the nonlocal game are preserved?

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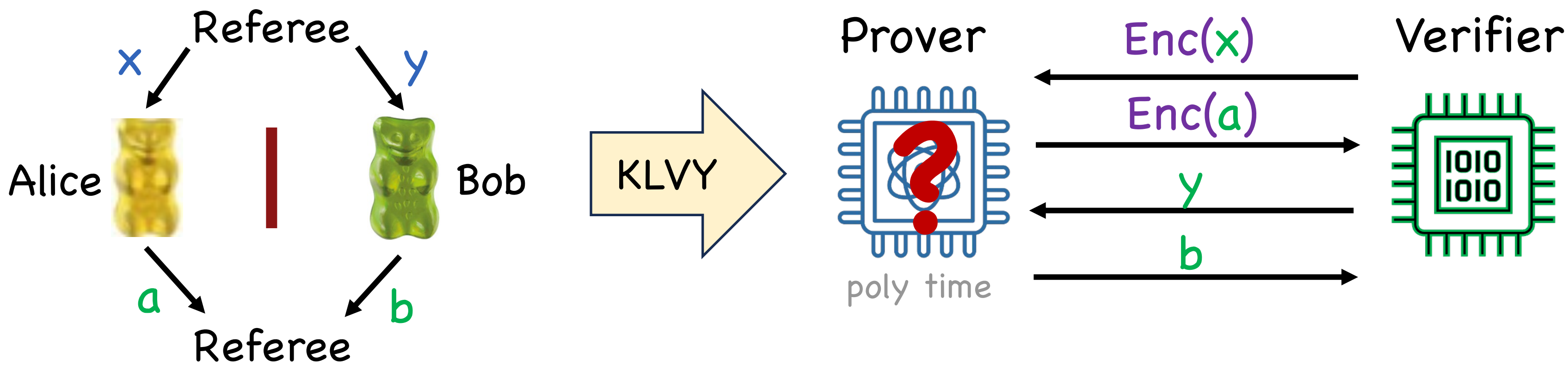


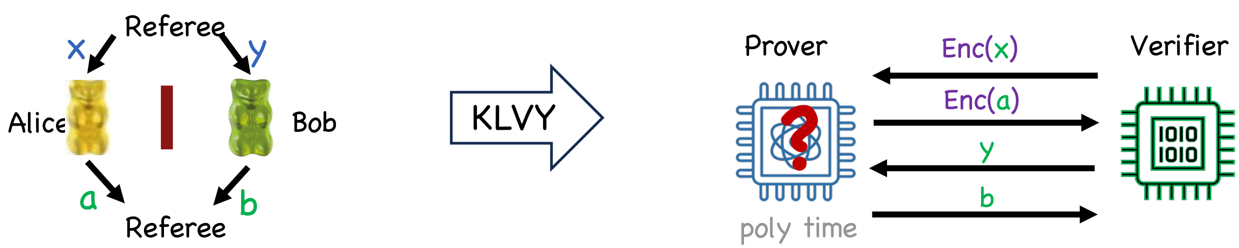
Key question: What properties of the nonlocal game are preserved?

As discussed, provers can do at least as well as in nonlocal game. $\omega_{\text{compiled}} \geq \omega$

But why can't they do better? Not obvious!

- At first glance, cryptography only ensures "non-signaling".
- But this is **not** enough!
- Natural variations do **not** work ("spooky" encryption)!

What we know: Trading space for time



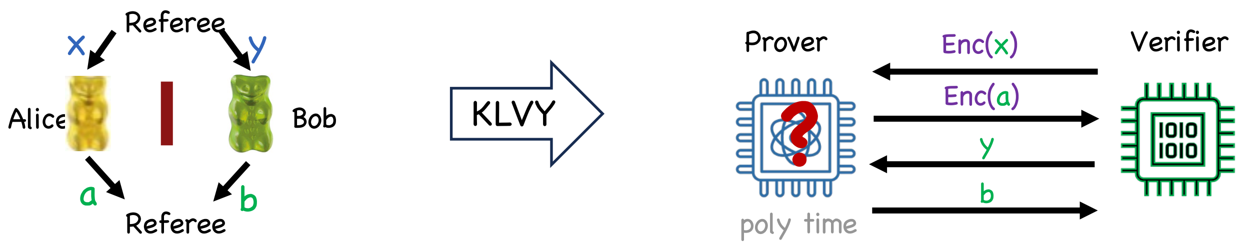
Classical Soundness (KLVY): Efficient classical provers cannot cheat, i.e. exceed classical value of nonlocal game.

$\omega_{c,\text{compiled}} \leq \omega_c$

Thus, if observe $p_{win} > \omega_c$ this constitutes proof of non-classicality! ☺

All results hold for large security parameter ("key length").

What we know: Trading space for time



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Computational Tsirelson Theorem (Natarajan-Zhang, Cui-…-W):

- For "XOR games", quantum provers cannot exceed q. value.
- Near optimal strategies yield "logical qubits" inside prover!

$\omega_{q,\text{compiled}} \leq \omega_q$

$$B_0 B_1 \approx -B_1 B_0$$

This is good enough to verify q. computations. ☺

All results hold for large security parameter ("key length").

What we know: Trading space for time for <u>all</u> nonlocal games

[Kulpe-Malavolta-Paddock-Schmidt-W]

XOR games are *special* - they don't probe full power & complexity of spacelike quantum correlations. What can we say about *general* games?

All results hold for large security parameter ("key length").

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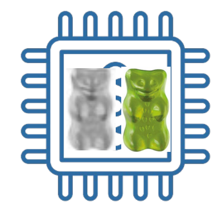

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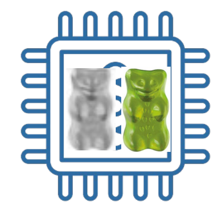


To prove these, we connect notions that are usually treated separately:

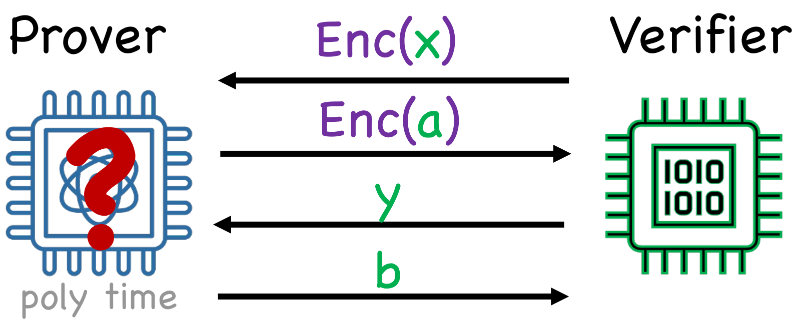
(1) *Timelike* characterization of *spacelike* correlations.
(2) *Computational* security → *info-theoretic* security.

Of independent interest?

How does it work?

Task: Given a quantum prover for the compiled game, wish to construct quantum strategy for the two-player game.

Let's analyze the situation:



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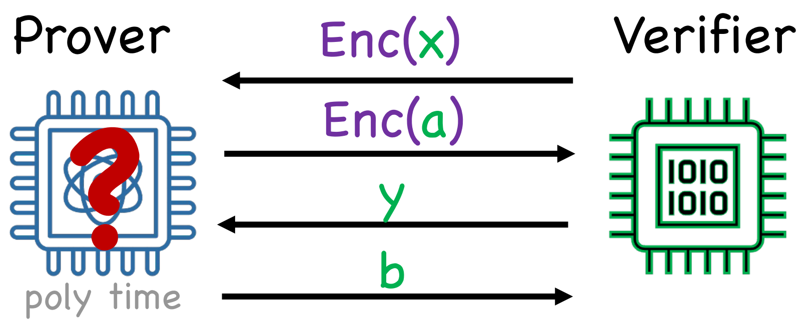

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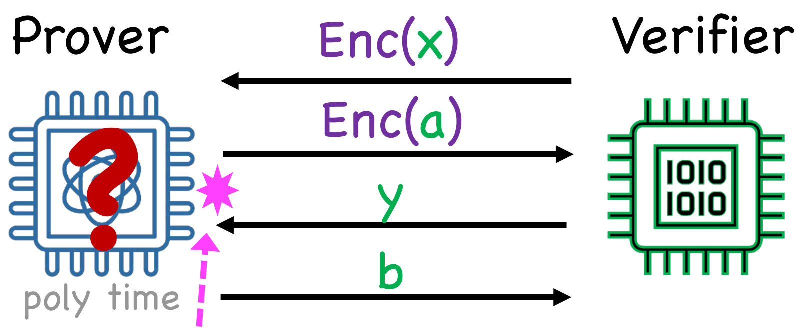

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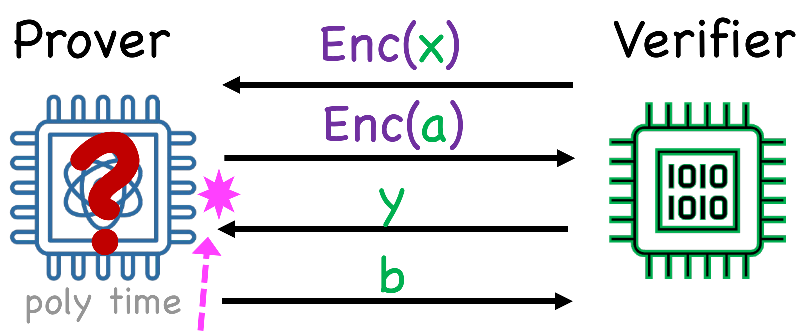

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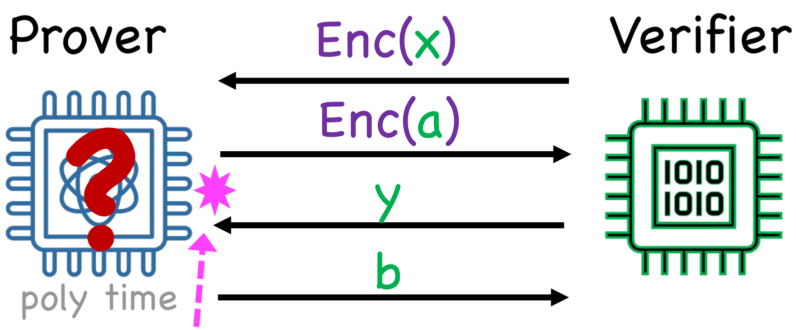

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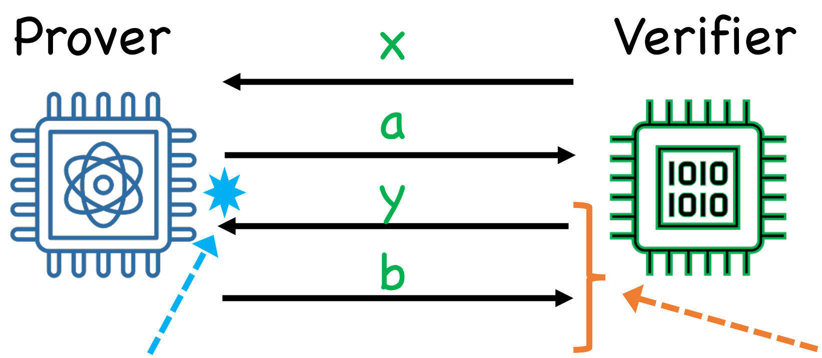
What if they were truly indistinguishable?

An information-theoretic toy model



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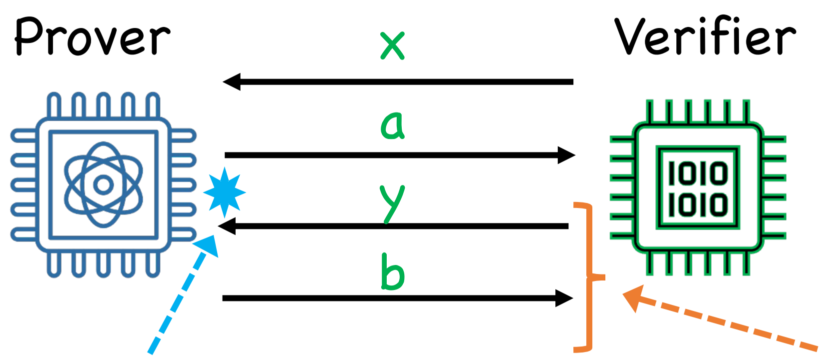

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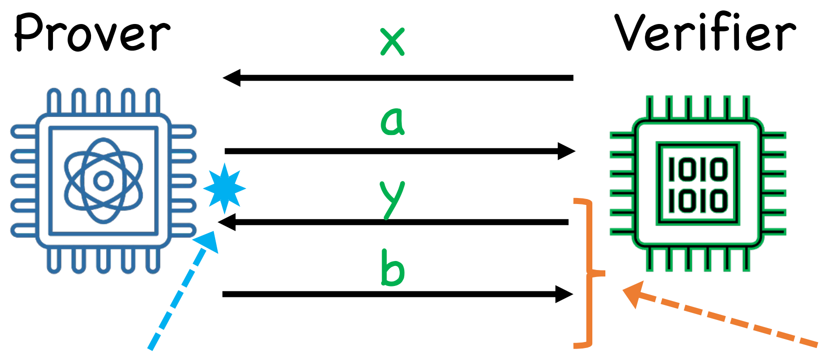
1st Ingredient: Timelike characterization of spacelike correlations



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Theorem: p(a,b|x,y) = $\rho_{x,a}(B_{y,b})$ is **quantum commuting op. correlation.**

In type I, an older result by [Navascues et al] shows that condition implies quantum $\otimes$ correlations. For us this unfortunately does not apply...

Connecting computational and information-theoretic security

$\lim_{\lambda \to \infty}$

Challenge: Intuitively, in compiled game have "strong non-signaling" for poly-time observables – but these don't form an algebra.

Moreover, $\rho_{x,a}^{\lambda}$ & $B_{y,b}^{\lambda}$ live on different (larger and larger) Hilbert spaces…

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Solution: Work with $\mathcal{B}$ = universal POVM algebra. This is an infinite dimensional C^*-algebra, but independent of security parameter!

Then we can define a sequence of states on the same algebra:

$$\varphi_{x,a}^{\lambda}\left(B_{y_1 b_1} B_{y_2 b_2} \ldots\right) := tr(\rho_{x,a}^{\lambda}\, B_{y_1 b_1}^{\lambda} B_{y_1 b_1}^{\lambda} \ \ldots)$$

2^{nd} Ingredient: Computational cryptography at infinite key length

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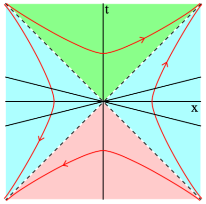
Theorem: For any quantum prover for the compiled game, limiting strategies at $\lambda = \infty$ exist and are strongly non-signaling! ☺

Proof uses quantum algorithmic techniques such as block encodings.

Open problems and speculations



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Both plausible! Surprisingly, *not* absurd to approximate commuting operator correlations by finite-dim. objects…

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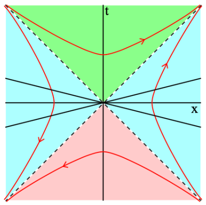

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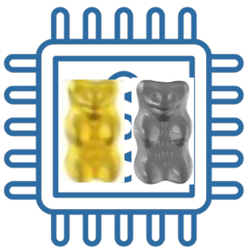

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Other situations in which one can connect computational and information-theoretic security?

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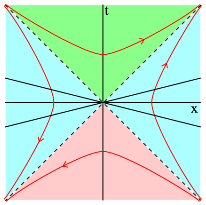

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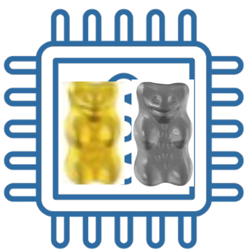

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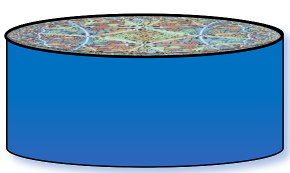
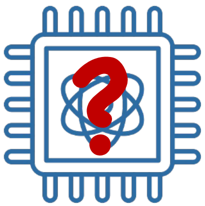


These results show rigorously how spacelike correlations can emerge from perspective of poly-time "observers". Anything to learn for quantum gravity?

Cf. works connecting complexity, pseudo-randomness, holography [Susskind-Maldacena, May, Bouland et al, …]

Summary

Nonlocal games are a foundational tool in quantum information and complexity.



Recent results establish links between the traditional space-like (information theoretic) and a time-like (computational) setting.

This gives new protocols to verify quantum advantage, computations, etc. It may also offer new insights into how locality can emerge in low-complexity effective theories.

Thank you for your attention!