Entanglement renormalization and CFT:

Quantum circuits for the Dirac field in 1+1 dimensions

Michael Walter

University of Amsterdam · QuSoft Research Center for Quantum Software · NWO

It from Qubit workshop, Kyoto, June 2019

with Freek Witteveen, Volkher Scholz, Brian Swingle (1905.08821)

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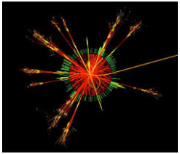
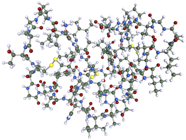
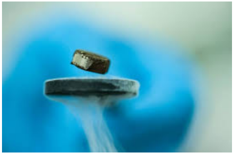
Complexity of many-body quantum physics

Many-body states have exponentially large description:

$$|\Psi\rangle = \sum_{i_1,\dots,i_n} \Psi_{i_1,\dots,i_n} \, |i_1,\dots,i_n\rangle$$

In practice, entanglement local $\rightsquigarrow$ compact description:

Start with local entangled pairs. . .

. . . and glue by applying local transformations:

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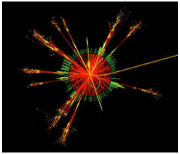
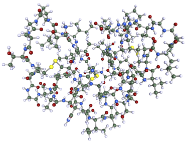
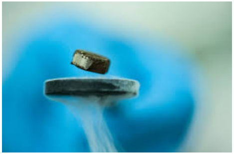


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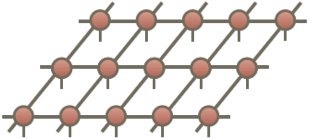
Tensor networks in practice and theory

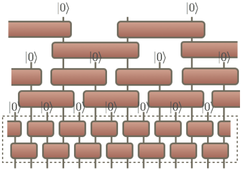
Tensor network: many-body state defined by contracting network of (local) tensors

matrix product state

[White, Fannes-Nachtergaele-Werner, Östlund-Rommer]



PEPS [Verstraete-Cirac]



MERA [Vidal]

Numerical tool: ansatz classes for many-body states

- geometrize entanglement: *area (RT) laws*
- some are *quantum circuits*

Powerful theoretical formalism that provides 'dual' or 'holographic' descriptions of complex phenomena: topological order, ...

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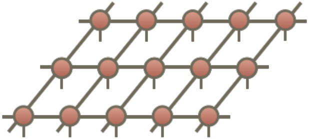

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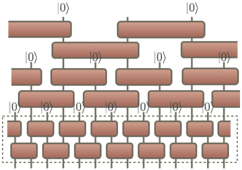

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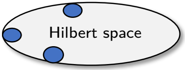

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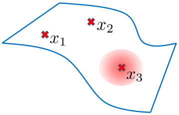
Tensor networks and quantum field theory

Quantum field theories are defined in the continuum, while tensor networks are discrete and finitary. How to reconcile?

Two successful approaches:

- modify ansatz $\rightsquigarrow$ continuum tensor networks
- relate discrete networks to correlation functions of continuum theory = unified perspective!



In either case. . .

What do tensor networks capture? Can we identify general construction principles? Why do tensor networks work well?

cf. plethora of rigorous results on gapped lattice systems in 1+1d [Hastings, . . .]

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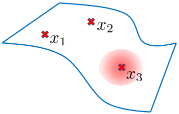

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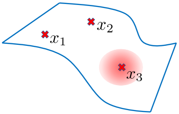

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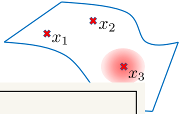


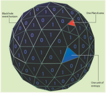
In ei

cf. pl

More generally: **How to study QI in QFT?**

☹ notions like *subsystem*, *entropy*, *approximation*, etc. subtle

☺ exciting recent progress. large body of work in mathematical physics.



c-theorem from subadditivity, Bekenstein bound via relative entropy, renormalization vs QEC, approximate QEC, . . .

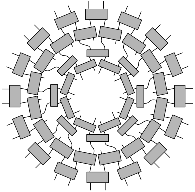
Our contributions

Result

We construct tensor networks for the Dirac CFT in 1+1 dimensions.

Key features:

- explicit construction – no variational optimization
- rigorous approximation of correlation functions
- quantum circuits that renormalize entanglement



We achieve this using tools from signal processing: multiresolution analysis and discrete/continuum duality from wavelet theory.

Also obtain sub-circuits for Majorana and Ising CFT. In prior work, we constructed (branching) MERA for critical free-fermion lattice models.

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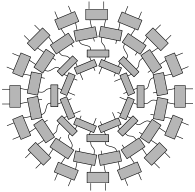

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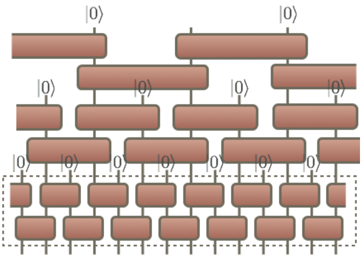
Tensor network ansatz for critical systems: [Vidal]

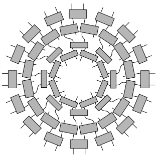


↓ local quantum circuit that prepares state from $|0\rangle^{\otimes N}$

↑ entanglement renormalization

↕ organize q. information by scale

- introduce entanglement at all scales / disentangle & coarse-grain
- noise-resilient on quantum computer [Kim et al]
- reminiscent of holography [Swingle], starting point for tensor network models [Qi, HaPPY, Hayden-...-W]



Important to understand design principles! Can we bridge numerics and toy models?

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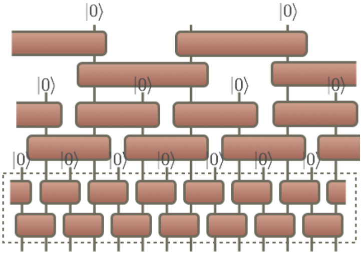

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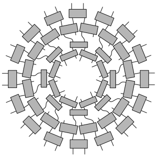

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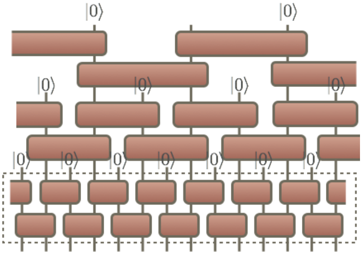

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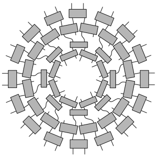

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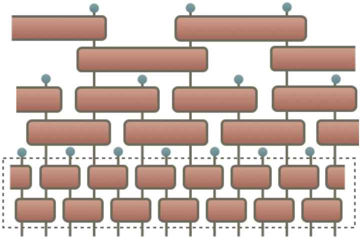

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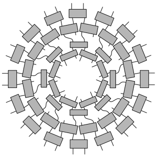

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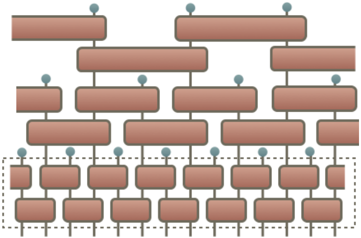

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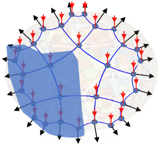

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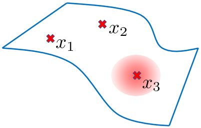
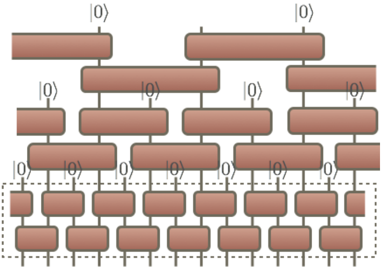
Result: Entanglement renormalization for Dirac CFT

Massless Dirac fermion in 1+1d: $i\gamma^\mu \partial_\mu \psi = 0$



We construct MERAs that target vacuum correlation functions $C = \langle O_1 \cdots O_n \rangle$ of smeared observables.



Result (simplified)

$$C_{\text{exact}} \approx C_{\text{MERA}}$$

Goodness depends on smearing, #layers, quality parameter. Comes with 'dictionary' for mapping observables. Symmetries approximately inherited!

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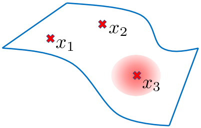

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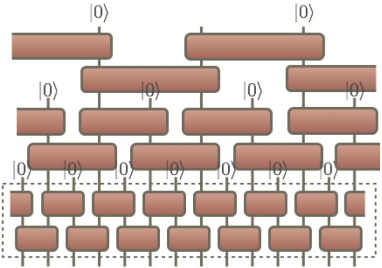

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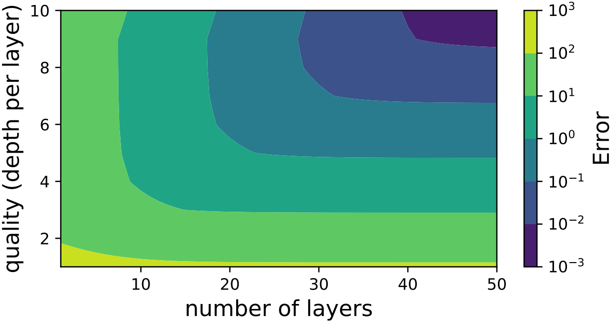
Two-point functions: A-priori error

For different values of quality parameter and number of layers:



Similarly for higher-point functions.

Two-point functions: Numerics

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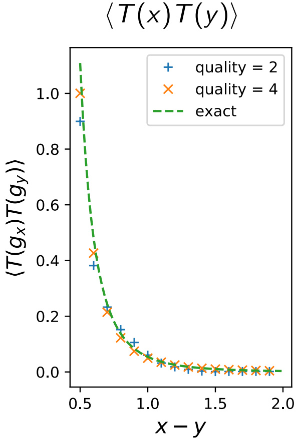
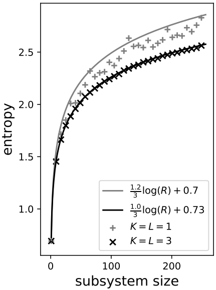


Verifying conformal data

- central charge: $S(R) = \frac{c}{3} \log R + c'$



- usual procedure: identify fields by searching for operators that coarse-grain to themselves

  $\rightsquigarrow$ diagonalize 'scaling superoperator' [Evenbly-Vidal]

	K=L=1	K=L=2
Δ_I	0	0
Δ_η	0.5	0.5
$\Delta_{\bar\eta}$	0.5	0.5
Δ_ε	1	1
Δ_σ	0.097	0.131
Δ_μ	0.170	0.120

(for Majorana subtheory)

- not necessary in order to compute correlation functions using our MERAs – theorem provides dictionary!

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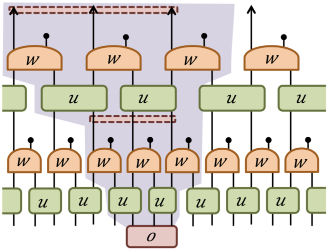

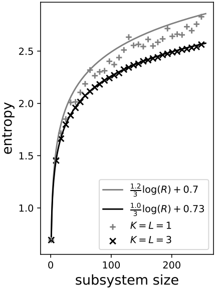

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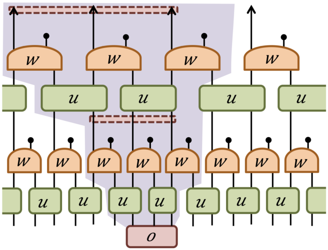

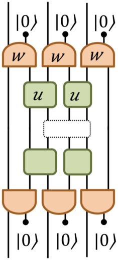

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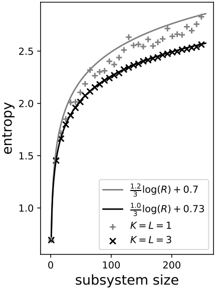

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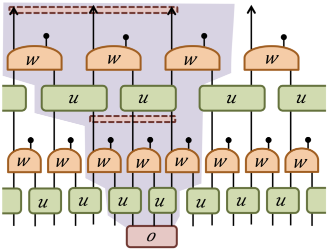

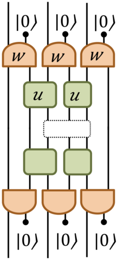

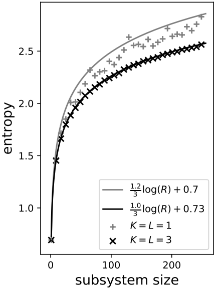

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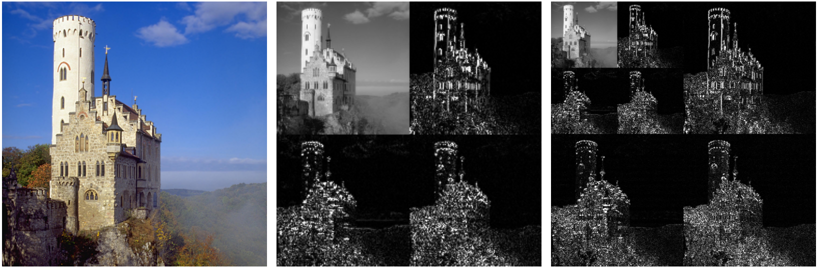
How does it work?

Key technique: Entanglement renormalization using wavelet theory.

Tool from signal processing to resolve signal by scale:



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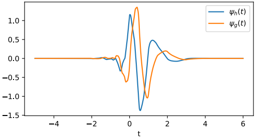

We construct wavelets that target Fermi sea of Dirac field using recent signal processing results (Selesnick’s Hilbert pairs).

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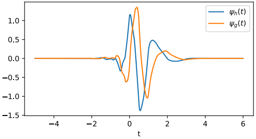
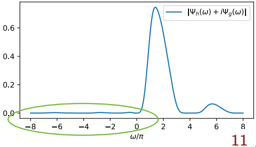


We construct wavelets that target Fermi sea of Dirac field using recent signal processing results (Selesnick's Hilbert pairs).



In more detail: Wavelets and MERA

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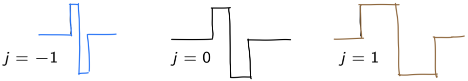

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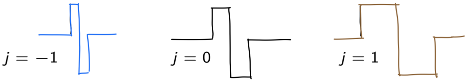

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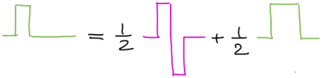

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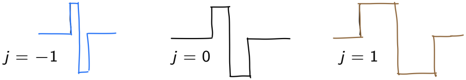

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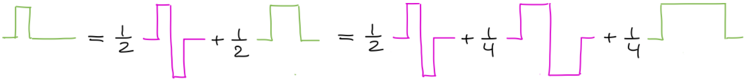

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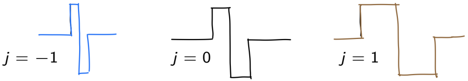

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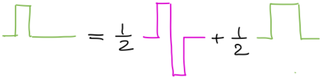

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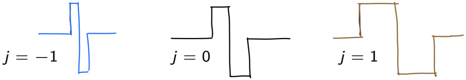

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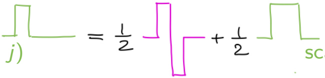

scaling basis (scale $\geq j{+}1$)

wavelet basis (scale $= j$)

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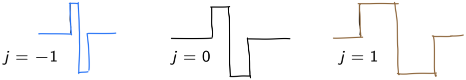

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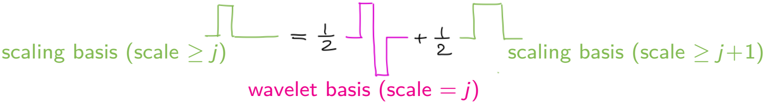

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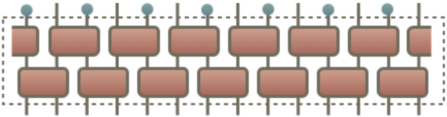

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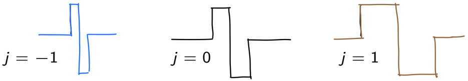

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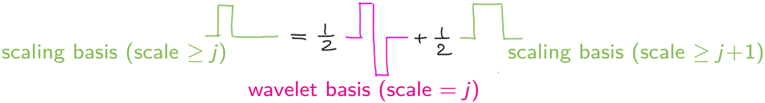

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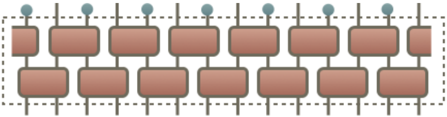

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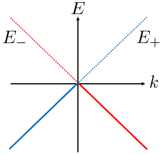
Wavelets for the Dirac fermion

Massless Dirac equation in 1+1d:

$$\begin{bmatrix} \psi_1(x,t) \\ \psi_2(x,t) \end{bmatrix} = \begin{bmatrix} \chi_+(x-t) + \chi_-(x+t) \\ \mathrm{i}\chi_+(x-t) - \mathrm{i}\chi_-(x+t) \end{bmatrix}$$



Need wavelets that target negative/positive momenta. Studied in signal processing, motivated by *directionality* and *shift-invariance*! [Selesnick]

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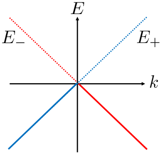

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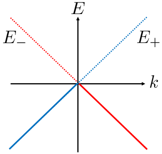

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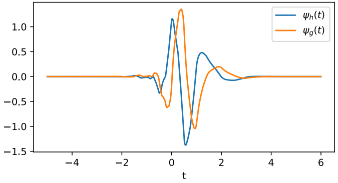

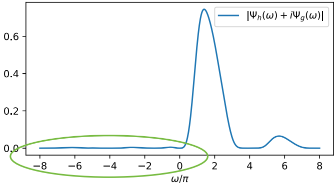

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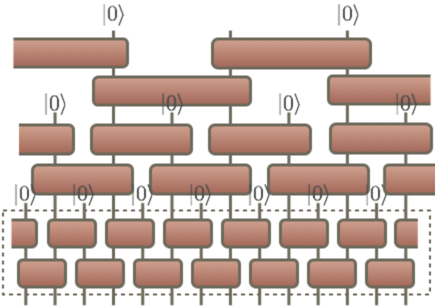
Technical Result



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- $\mathcal{L}$ – number of layers
- ε – accuracy of wavelet pair
- Γ – support and smoothness of smearing functions

Consider correlation function with smeared fields & normal-ordered bilinears:

$$C := \langle \Psi^\dagger(f_1) \cdots \Psi(f_{2N}) O_1 \cdots O_M \rangle$$

Theorem (simplified)

$$|C_{\text{exact}} - C_{\text{MERA}}| \leq \Gamma \max\{2^{-\mathcal{L}/3}, \varepsilon \log \frac{1}{\varepsilon}\}$$

We provide dictionary for C_{MERA} (discretize smearing functions in scaling basis etc).

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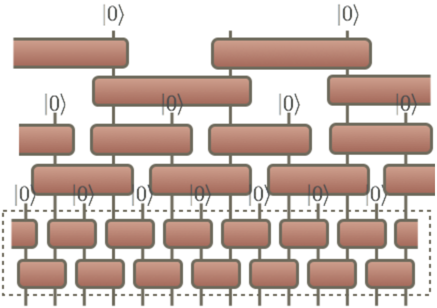

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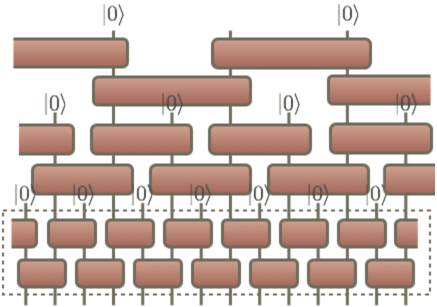

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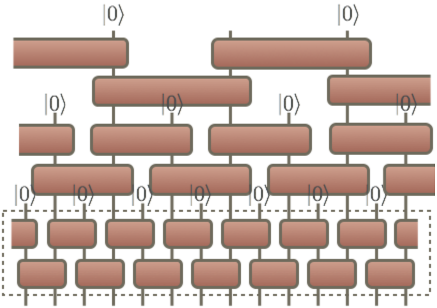

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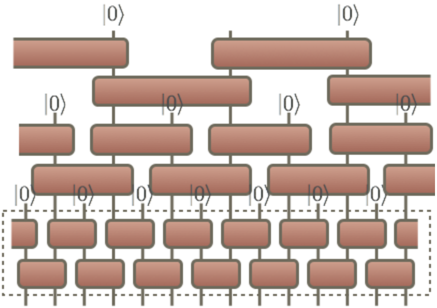

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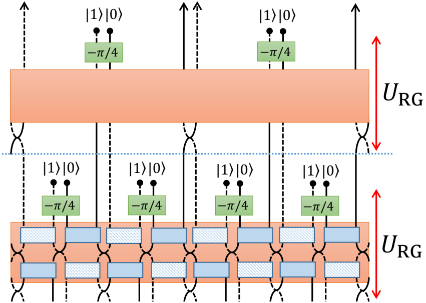

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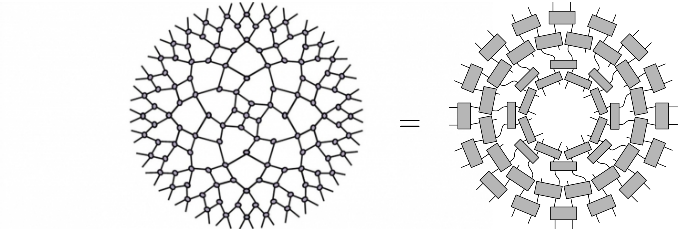
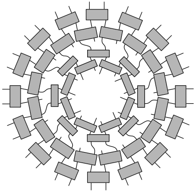
Dirac fermion on circle

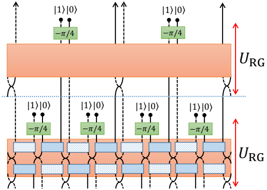
Construction can be easily adapted to Dirac fermion on circle:



- finite number of layers once UV cut-off fixed
- systematic construction by (anti)periodizing wavelets

Summary and outlook





- entanglement renormalization quantum circuits for 1+1d Dirac CFT
- systematic construction with rigorous guarantees

Outlook:

- thermofield double, Dirac cones, ...
- building block for more interesting CFTs? starting point for perturbation theory or variational optimization?
- lift wavelet theory to quantum circuits! tensor network bootstrap?

Thank you for your attention!

How to build an approximate Hilbert pair [Selesnick]

Wavelets are built from filters $g[n]$ that relate functions at different scale:

$$\phi_{j-1}(x) = \sum_{n \in \mathbb{Z}} g[n] \, \phi_j(x - 2^{-j} n)$$

Necessary and sufficient to obtain orthonormal basis (roughly speaking):

$$|G(\theta)|^2 + |G(\theta + \pi)|^2 = 2, \quad G(0) = \sqrt{2}$$

Wavelets are related by Hilbert transform iff filters related by half-shift:

$$G(\theta) = H(\theta) e^{-i\theta/2}$$

To achieve this, find explicit approximation

$$e^{-i\theta/2} \approx e^{-iL\theta} \frac{D(-\theta)}{D(\theta)}.$$

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Non-relativistic 2D fermions – Lattice model

When put on lattice, massless Dirac fermion becomes: (Kogut-Susskind)

$$H_{1D} \cong -\sum_n a_n^\dagger a_{n+1} + h.c.$$

Non-relativistic fermions hopping on 2D square lattice at half filling:

$$H_{2D} = -\sum_{m,n} a_{m,n}^\dagger a_{m+1,n} + a_{m,n}^\dagger a_{m,n+1} + h.c.$$

Fermi surface:

- violation of area law: $S(R) \sim R \log R$ (Wolf, Gioev-Klich, Swingle)
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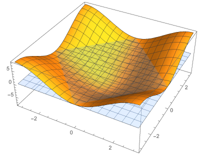

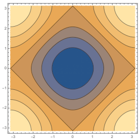

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Non-relativistic 2D fermions – Branching MERA

Natural construction – perform wavelet transforms in both directions:

$$W\psi = \psi_{\text{low}} \oplus \psi_{\text{high}} \quad \rightsquigarrow \quad (W \otimes W)\psi = \psi_{ll} \oplus \psi_{lh} \oplus \psi_{hl} \oplus \psi_{hh}$$

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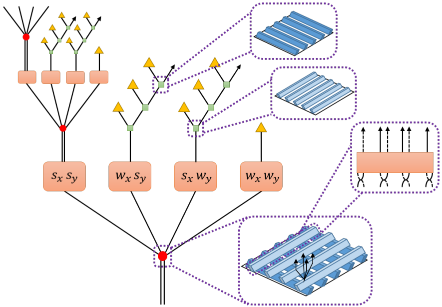

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