Quantum Information Theory, Spring 2020

Homework problem set #10 **due April 13, 2020**

Rules: Always explain your solutions carefully. You can work in groups, but must write up your solutions alone. You must submit your solutions before the Monday lecture (in person or by email).

1. *(2 points)* **Discriminating Bell states by LOCC:** Recall that the Bell states are given by

$$|\Phi^{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \qquad |\Phi^{01}\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle),$$
$$|\Phi^{10}\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle), \qquad |\Phi^{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle).$$

Assume that Alice holds the first qubit of a Bell state and Bob holds the second qubit.

(a) Find an LOCC protocol that can perfectly discriminate between $|\Phi^{00}\rangle$ and $|\Phi^{01}\rangle$.
(b) Find an LOCC protocol that can perfectly discriminate between $|\Phi^{00}\rangle$ and $|\Phi^{10}\rangle$.

2. *(6 points)* **One-way LOCC struggle:** Let $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$. Consider a two-qubit system where Alice holds the first qubit and Bob holds the second qubit. These two qubits are initialized in one of the following four states:

$$|\Psi_1\rangle = |0\rangle \otimes |0\rangle,$$
$$|\Psi_2\rangle = |0\rangle \otimes |1\rangle,$$
$$|\Psi_3\rangle = |1\rangle \otimes |+\rangle,$$
$$|\Psi_4\rangle = |1\rangle \otimes |-\rangle.$$

(a) Show that if μ is a separable measurement, and μ perfectly distinguishes an orthonormal basis, then this basis must consist of product states.
(b) Write down a measurement that perfectly distinguishes the above four states and show that it is separable.
(c) Find a one-way LOCC measurement from Alice to Bob that perfectly determines which of the four states they share.
(d) Show that there is no one-way LOCC measurement from Bob to Alice that can perfectly determine which of the four states they share.
Hint: Show that the choice of measurement for Alice can not depend on the outcome of Bob if she wants to perfectly distinguish the remaining states on her qubit.

3. *(4 points)* **Operations on PPT states:** Recall from Corollary 9.9 that a state $\rho_{AB} \in D(\mathcal{H}_A \otimes \mathcal{H}_B)$ has positive partial transpose (PPT) if $(\mathcal{T}_A \otimes \mathcal{I}_B)[\rho_{AB}] \geqslant 0$ where $\mathcal{T}[X] = X^T$ is the transpose map. Suppose that Alice and Bob share such PPT state.

(a) Show that if they apply a separable channel Ξ, the resulting state $\Xi[\rho_{AB}]$ is again PPT.
(b) Show that they cannot get a maximally entangled state

$$|\Phi^+_{AB}\rangle = \frac{1}{\sqrt{|\Sigma|}} \sum_{a \in \Sigma} |a\rangle_A \otimes |a\rangle_B$$

with any $|\Sigma| > 1$ by applying an LOCC operation on a PPT state ρ_{AB}.