Last time: Entanglement entropy of $|\psi\rangle_{AB}$:

$$S_E(\psi) := S(\rho_A) = S(\rho_B)$$

① (optimal) quantum compression rate

OPERATIONAL INTERPRETATION

② $S_E = E_C = E_D$:

$$\text{"}\ |\psi\rangle_{AB}^{\otimes n} \underset{\longleftarrow}{\xrightarrow{\text{LOCC}}} |\Phi_2^+\rangle^{\otimes n \cdot S_E(\psi)}\ \text{"}$$

via

$$S_E \geq E_C \geq E_D \geq S_E$$

"ent. concentration"

STILL NEED TO PROVE

Entanglement dilution

$$|\Phi_2^+\rangle^{\otimes R \cdot n} \xrightarrow{\text{LOCC}} \approx |\psi\rangle_{AB}^{\otimes n}$$

Idea:

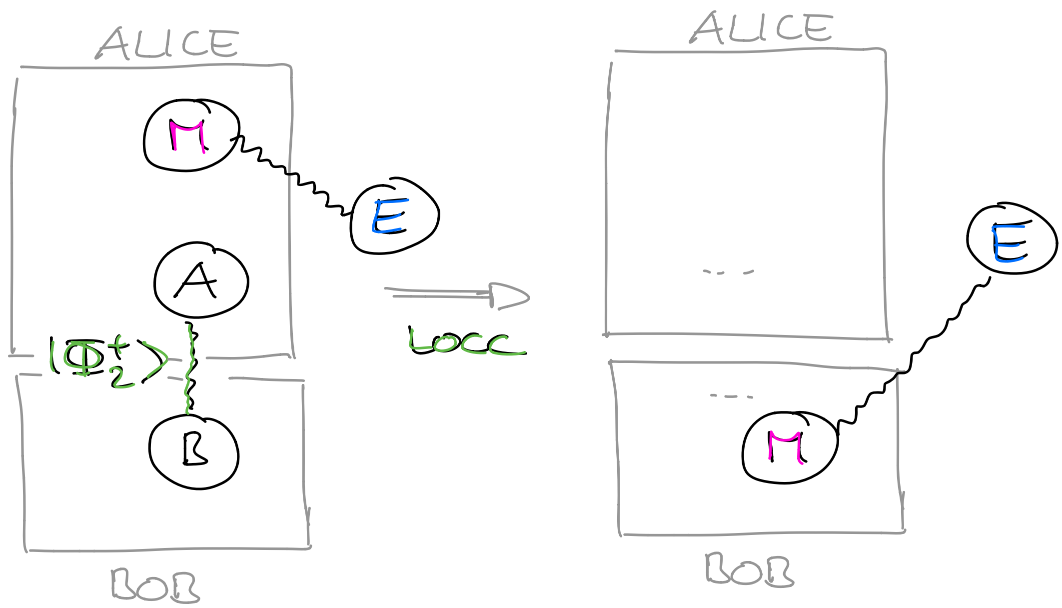
- Alice can prepare $|\psi\rangle_{AB}^{\otimes n}$ in her laboratory.
- Q. compression allows to transfer B-systems to Bob by sending $\approx n \cdot (S_E + \delta)$ qubits

Q: Can we instead use ebits and LOCC?

Quantum Teleportation

Alice and Bob share ebit $|\Phi_2^+\rangle_{AB}$

Goal: Send qubit M of Alice over to Bob.



$$\underbrace{|\psi\rangle_{EM} \otimes |\Phi_2^+\rangle_{A}}_{\text{Alice}}\underbrace{{}_{B}}_{\text{Bob}} \qquad \cdots \otimes |\psi\rangle_{E\underbrace{M}_{\text{Bob}}}$$

(Entanglement of M should be preserved.)

- No cloning: Can only succeed if Alice learns nothing about state of M (no info remains in her lab)
- OTOH: Alice should couple A & M.

↳ measure AM in basis of max entangled states:

$$|\phi_0\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) = (I \otimes I)\,|\Phi_2^+\rangle$$

$$|\phi_1\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right) = (I \otimes Z)\ |\Phi_2^+\rangle$$

$$|\phi_2\rangle = \frac{1}{\sqrt{2}}\left(|10\rangle + |01\rangle\right) = (I \otimes X)\ |\Phi_2^+\rangle$$

$$|\phi_3\rangle = \frac{1}{\sqrt{2}}\left(|10\rangle - |01\rangle\right) = (I \otimes XZ)\,|\Phi_2^+\rangle$$

i.e. $|\phi_k\rangle_{AM} = (I \otimes U_k)\,|\Phi_2^+\rangle_{AM}$

↳ proj. measurement $P_{AM,k} = |\phi_k\rangle\langle\phi_k|_{AM}$

Pr(outcome k)

$$= \operatorname{tr}\left[P_{AM,k} \cdot \operatorname{tr}_{EB}\left[|\psi\rangle\langle\psi|_{EM} \otimes |\Phi_2^+\rangle\langle\Phi_2^+|_{AB}\right]\right]$$

$$= \operatorname{tr}\left[|\phi_k\rangle\langle\phi_k|_{AM} \cdot \left(\operatorname{tr}_E\left[|\psi\rangle\langle\psi|_{EM}\right] \otimes \frac{I_A}{2}\right)\right]$$

$$= \frac{1}{2}\operatorname{tr}\left[\operatorname{tr}_A\left[|\phi_k\rangle\langle\phi_k|_{AM}\right] \cdot \operatorname{tr}_E\left[|\psi\rangle\langle\psi|_{EM}\right]\right]$$

$$= \frac{1}{2}\operatorname{tr}\left[\frac{I_M}{2}\operatorname{tr}_E\left[|\psi\rangle\langle\psi|_{EM}\right]\right] = \frac{1}{4}.$$

COMPLETELY UNINFORMATIVE :)

Post-measurement state?

$$2\left(\langle\phi_k|_{AM} \otimes I_{EB}\right)\left(|\psi\rangle_{EM} \otimes |\Phi_2^+\rangle_{AB}\right)$$

$$= 2\left[I_E \otimes \underbrace{\left(\langle\Phi_2^+|_{AM} \otimes I_B\right)\left(I_M \otimes |\Phi_2^+\rangle_{AB}\right)}\right]$$

$$= ? \qquad \left[I_E \otimes U_{M,k}^\dagger\right]|\psi\rangle_{EM}$$

$$\left(\langle\Phi_2^+|_{AM}\otimes I_B\right)\left(I_M\otimes|\Phi_2^+\rangle_{AB}\right)$$

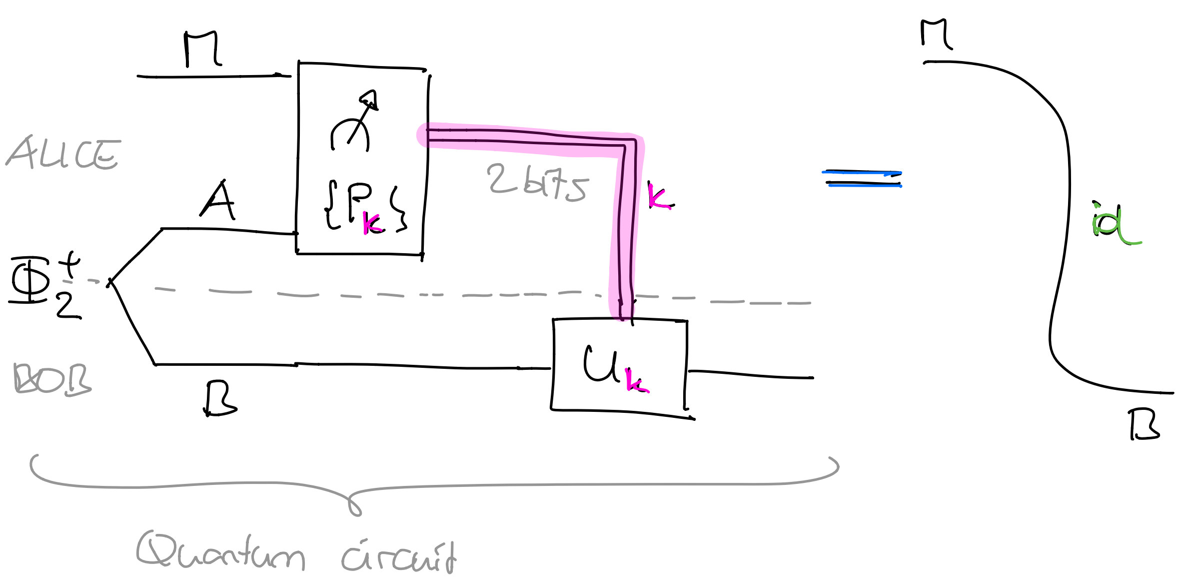
$$= \frac{1}{2}\sum_{x,y}\left(\langle x|_A\otimes\langle x|_M\otimes I_B\right)\left(|y\rangle_A\otimes I_M\otimes|y\rangle_B\right)$$

$$= \frac{1}{2}\boxed{\sum_x |x\rangle_B\langle x|_M}$$

IDENTITY FROM M TO B

$$\Longrightarrow \ ? = \left(I_E\otimes U_{B_1}^{\dagger}\right)|\psi\rangle_{EB}$$

Alice sends over k & Bob applies U_{B_1k}: SUCCESS



- LOCC! Zero error, non-asymptotic
- Composable: n ebits $\overset{LOCC}{\leadsto}$ n qubits

↳ Can convert <u>any</u> compression protocol into entanglement dilution protocol at same rate

$$\Longrightarrow \quad E_S \geq R^{opt.}_{Compr} \geq E_C \geq E_D \geq E_S$$

compression protocol from L8

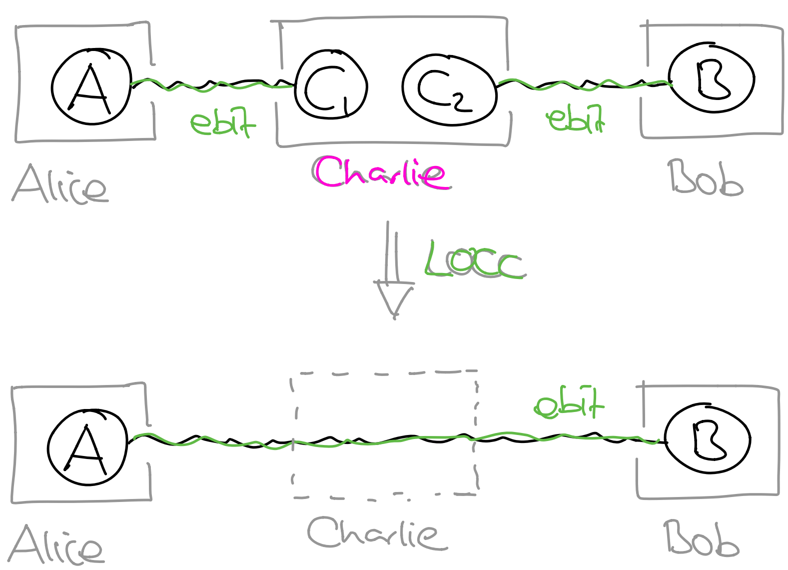
JUST PROVED

ent. concentration from L8

$$\Longrightarrow \quad \boxed{E_S = R^{opt}_{Compr.} = E_C = E_D}$$

everything we wanted to prove

<u>Entanglement swapping:</u>



→ Delft experiment

• Similarly with many intermediate "relays"!

Resource inequalities

Teleportation:

$$\text{ebit} + 2\underbrace{[c \to c]}_{\text{1 bit of class. commun.}} \overset{\text{Local Operations}}{\geq} \underbrace{[q \to q]}_{\text{1 qubit of quantum communication}}$$

What else can we do?

- $[q \to q] \geq \text{ebit}$ ← Alice prepares ebit, sends over half of it

BUT: $\text{ebit} \ngeq [q \to q]$ CANNOT COMMUNICATE USING ENTANGLEMENT ALONE

- $[q \to q] \geq [c \to c]$ ← Alice encodes x by $|x\rangle$, Bob measures $\{|x\rangle\langle x|\}$

BUT: $[q \to q] \ngeq 2[c \to c]$ "HOLEVO BOUND"

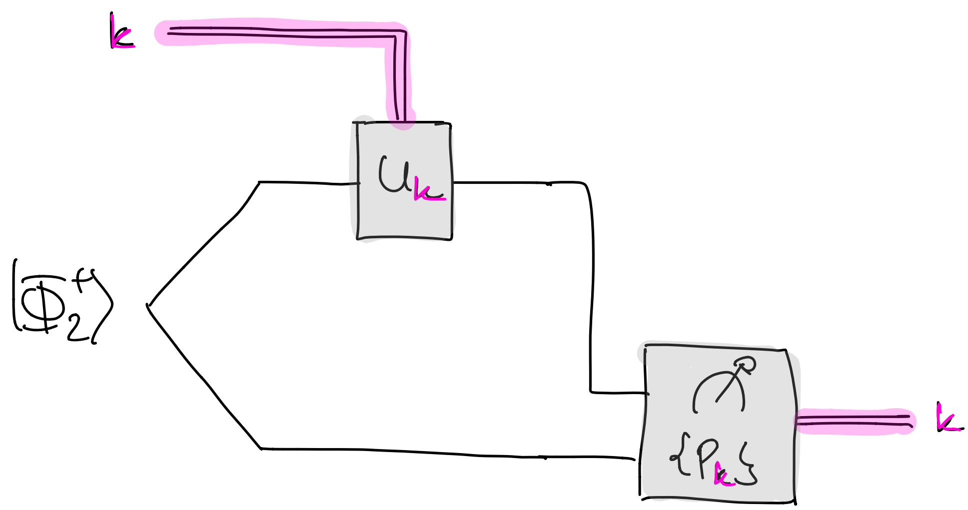
Superdense coding:

(not quite) converse of teleportation

$$[q \to q] + \text{ebit} \geq 2[c \to c]$$

$$\Longrightarrow \boxed{[q \to q] \equiv 2[c \to c] \mod \text{ebit}}$$

Protocol:
- Alice and Bob share $|\Phi_2^+\rangle_{AB}$
- Alice applies some U_k to A & sends A over to Bob
- Bob measures $\{P_k\}$



Resource inequalities are very useful to relate QIP protocols in their strength.