

# Introduction to Information Theory, Fall 2019

## Practice problem set #11

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You do **not** have to hand in these exercises, they are for your practice only.

1. **Suboptimality of symbol codes:** Recall that the optimal symbol code  $C$  for a source  $X$  satisfies

$$H(X) \leq L(C, X) < H(X) + 1.$$

Show that for any  $\varepsilon > 0$  there exists a source such that for the optimal symbol code  $C$

$$L(C, X) > H(X) + 1 - \varepsilon.$$

*Hint: construct an example for a binary source.*

2. **Entropy and probability:** Suppose  $X$  and  $Y$  are independent random variables taking values on the same alphabet with respective distributions  $P$  and  $Q$ . Show that

$$\Pr(X = Y) \geq 2^{-H(P) - D(P||Q)}.$$

*Hint: Jensen's inequality.*

3. **Channel capacity** Suppose we have a channel  $Q(y_1, y_2|x) = Q(y_1|x)Q(y_2|x)$ . That is, for any fixed input symbol  $x$ , the output distribution is IID. Now let  $X$  be a random channel input and denote by  $Y = (Y_1, Y_2)$  the corresponding channel output.

- (a) Show that

$$I(X : Y) = 2I(X : Y_1) - I(Y_1 : Y_2).$$

- (b) Show that the capacity of the channel  $X \rightarrow Y$  is at most twice the capacity of the channel  $X \rightarrow Y_1$ .