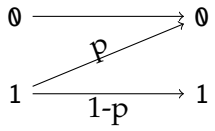
Introduction to Information Theory, Fall 2019

Homework problem set #1 **due September 13, 2019**

Rules: Always explain your solutions carefully. You can work in groups, but must write up your solutions alone. You must submit your solutions before the Friday exercise class (either in person or by email).

1. **Binary asymmetric channel (1 point):** Imagine a channel that always transmits `0` correctly, but which flips 1 with probability p. You can visualize this as follows:



   Suppose that you would like to communicate a uniformly random bit by using this channel.

   (a) When using the channel directly, what is the average probability that the bit arrives flipped?
   (b) Now encode your bit using the repetition code R_3. What is the optimal decoder? What is the average probability that the bit arrives flipped when using the optimal decoder?

2. **(7,4)-Hamming code (1 point):**

   (a) Encode the bitstring $s = \texttt{1010}$ using the Hamming code.

   Now consider the following two bitstrings:

$$r_A = \texttt{1110100}, \quad r_B = \texttt{0110101}$$

   (b) Compute the parity bits $z = z_1 z_2 z_3$ as discussed in class.
   (c) Only one of the two bitstrings is a codeword of the Hamming code. Which one?
   (d) Decode both bitstrings using the decoder discuss in class.

3. **Simulating the repetition code (1 point):**
   In this problem, you may simulate the *binary symmetric channel* and the *repetition code* R_3 discussed in class. Your goal is to obtain a result similar to Figure 1.11 in MacKay's book. To get started, open the Python notebook at https://colab.research.google.com/github/amsqi/iit19-homework/blob/master/01-homework.ipynb and follow the instructions. While we tried to make everything self-explanatory, please do not hesitate to ask if anything is unclear!

   Please submit your solution as a Python notebook or script (*File* → *Download .ipynb / .py*), or as a PDF printout (*File* → *Print*). You can score the maximum score if your solution produces the correct output – we will only have a closer look at your code in case of problems.