

Introduction to Information Theory, Fall 2021

Homework problem set #6

due Dec 12, 2021

Rules: Always explain your solutions carefully. Please hand in the assignment in groups on Canvas. In the werkcollege the TAs can tell you more about how this works.

1. **Reed-Solomon codes (1 point):** Consider the Reed-Solomon code with the following parameters: $K = 2$, $N = 4$, $q = 7$, and $\alpha = 3$.
 - (a) Encode a general message $s^K = [s_1, s_2]$ using the encoding algorithm from class.
 - (b) Suppose that you receive $y^N = [0, 5, \perp, \perp]$, where \perp indicates an *erasure error*. Decode the message using the decoding algorithm from class.

2. **Distance & Singleton bound (1 point):** Let $\mathcal{A} = \{0, \dots, q - 1\}$. Given two strings x^N and \tilde{x}^N in \mathcal{A}^N , define their *Hamming distance* $d(x^N, \tilde{x}^N)$ to be the number of places in which x^N and \tilde{x}^N differ. Now suppose that $\mathcal{C} \subseteq \mathcal{A}^N$ is the set of codewords of an error-correcting code. We define the *distance* of the code to be

$$d := \min_{x^N \neq \tilde{x}^N \in \mathcal{C}} d(x^N, \tilde{x}^N).$$

- (a) Relate the distance to the number of *erasure errors* that can be corrected by the code.
- (b) Prove the following formula, known as the *Singleton bound*. It bounds the number of codewords in terms of the alphabet size, block size, and distance:

$$\#\mathcal{C} \leq q^{N-d+1}.$$

Hint: Erase as many symbols as possible without changing the number of codewords.

- (c) Deduce that the Singleton bound is saturated for the Reed-Solomon codes from class.

3.  **Reed-Solomon codes (1 point):**

In this problem, you will implement the encoding and decoding algorithms for Reed-Solomon codes. To get started, open the notebook at <https://colab.research.google.com/github/amsqi/iit21-homework/blob/master/06-homework.ipynb> and follow the instructions.

Please submit both the notebook and a PDF printout, or provide a link to your solution on Colab. You can achieve the maximum score if your solution produces the correct output. We will only have a closer look at your code in case of problems.