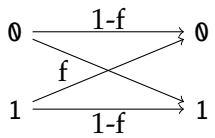
Introduction to Information Theory, Fall 2021

Homework problem set #1 **due Nov 7, 2021**

Rules: Always explain your solutions carefully. Please hand in the assignment in groups on Canvas. In the werkcollege the TAs can tell you more about how this works.

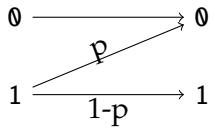
1. **Binary symmetric channel (1 point):** Recall the binary symmetric channel from class, which transmits a given bit correctly with probability $1 - f$ and flips the bit with probability f:



   Suppose we communicate a random bit over this channel by using the repetition code R_3. We can model this situation by three random variables: S is the message (the random bit) that we wish to communicate, $X = X_1X_2X_3$ is the codeword that we send over the channel (bit by bit), and $Y = Y_1Y_2Y_3$ is the signal that we receive on the other side. For concreteness, suppose S is 0 with probability $p = 1/4$, and the probability of flipping the bit is $f = 1/3$.

   (a) Make a table that contains the joint probability distribution of S and Y, as well as their marginal distributions. Your table should look like the one that we made in Lecture 2.
   (b) Are the first two bits of Y independent from each other?
   (c) Suppose you receive 010. What are the optimal decoding and the probability of error?

2. **Binary asymmetric channel (1 point):** Imagine a channel that always transmits 0 correctly, but which flips 1 with probability p. You can visualize this as follows:



   Suppose that you would like to communicate a uniformly random bit by using this channel.

   (a) When using the channel directly, what is the probability that the bit arrives flipped?
   (b) Now encode your bit using the repetition code R_3. What is the optimal decoder? What is the probability of error when using the optimal decoder?

3. **Simulating the repetition code (1 point):**
   In this problem, you will simulate the *binary symmetric channel* and the *repetition code* R_3 discussed in class and above. Your goal is to obtain a result similar to Figure 1.11 in MacKay's book. To get started, open the Python notebook at https://colab.research.google.com/github/amsqi/iit21-homework/blob/master/01-homework.ipynb and follow the instructions. We tried to make everything self-explanatory, but please do not hesitate to ask if anything is unclear!

   Please submit both the notebook (*File* → *Download .ipynb*) **and** a PDF printout (*File* → *Print*) of your solution. You can achieve the maximum score if your code produces the correct output – we will only have a closer look if it does not.