Introduction to Information Theory, Fall 2020

Practice problems for exercise class #1

You do **not** have to hand in these exercises, they are for your practice only.

0. **Exercises from MacKay:** 1.3, 1.5, 1.6, 1.7, 1.10

1. **Optimality of repetition code decoder:** We wish to communicate a bit $\in \{0, 1\}$ over a binary symmetric channel with noise level $f < 0.5$. To allow for error detection and correction, we use the repetition code R_3. That is, we send the bit over the channel *three times*: $x_1x_2x_3 = sss$. Suppose that s is uniformly random and that the receiver receives the signal $y_1y_2y_3 \in \{0, 1\}^3$.

   Show that the optimal way for the receiver to decode the message is by 'majority vote': if there are more 0s then 1s among $y_1y_2y_3$ then she interprets the message as 0, otherwise as 1. That is, the messages $000, 001, 010, 100$ are decoded as 0, and the other messages as a 1.