Introduction to Information Theory, Fall 2019

Practice problem set #10

You do **not** have to hand in these exercises, they are for your practice only.

1. **Reed-Solomon practice:** Consider the Reed-Solomon code from the lecture with parameters $K = 1$, $N = 3$, $q = 5$ and $\alpha = 2$.

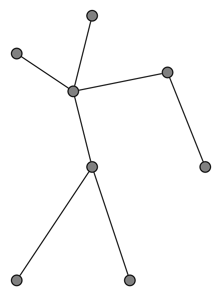
   (a) Suppose we receive $y^N = [2, 1, \perp]$. Fix the erasure error and decode the message.
   (b) Suppose we receive $y^N = [1, 1, 2]$. Fix the error (if any) and decode the message.

2. **Reed-Solomon decoding algorithm:** Write out the Reed-Solomon decoding algorithm for the case of $C \leqslant \lceil \frac{T}{2} \rceil$ errors at unknown locations in pseudocode.

   Hint: Follow the procedure outlined in the lecture.

3. **Message passing for counting vertices of graphs:** Recall that in the lecture we described a message passing algorithm for decentralized counting.

   (a) Describe how you could generalize the message passing algorithm to counting the number of vertices of graphs without cycles (these are called *trees*).
   (b) Work out explicitly what this algorithm would do for the graph below. Which node outputs the final answer?



   (c) Why does the algorithm fail in the presence of cycles? Find a concrete counterexample.

4. **Message passing for minimal cost paths:** Suppose we have a grid with two distinct nodes A and B and, as in the lecture, we can only walk steps down or to the right. Now we assume that each step has a cost associated to it, and we want to find the path from A to B with minimal total cost.

   (a) What is the separation property we can use?
   (b) Can you design a *forward* message passing algorithm that computes the minimal cost of a path between A and B?
   (c) Explain how you can use a backward pass to also obtain a minimal path (and not just the minimal cost).